6. Trigonometric Identities

Exercise 6.1

1. Question

Prove the following trigonometric identities:

$(1-\cos^2 A)\cos ec^2 A = 1$

Answer

$To\ Prove: (1 - \cos^2 A)\ \text{cosec}^2 A = 1$

$Proof:$

$1 - \cos^2 A = \sin^2 A$

$Therefore,\ L.H.S = \sin^2 A\ .\ \text{cosec}^2 A$

$Now,\ \text{cosec}^2 A = \frac{1}{\sin^2 A}$

$Therefore,\ L.H.S = \sin^2 A\ .\ \frac{1}{\sin^2 A} = 1$

$R.H.S = 1$

$L.H.S = R.H.S$

$Hence,\ Proved$

2. Question

Prove the following trigonometric identities:

$(1+\cot^2 A)\sin^2 A = 1$

Answer

Consider,

$(1+\cot^2 A)\sin^2 A$

As we know $1+\cot^2 A = \text{cosec}^2 A$

Putting the values we get,

$(\text{cosec}^2 A)\sin^2 A$

As we know,cosec A = 1/sinA

So,

$$\Rightarrow \frac{1}{\sin^2 A} \times sin^2 A = 1$$

hence proved

3. Question

Prove the following trigonometric identities:

$\tan^2\theta\cos^2\theta = 1-\cos^2\theta$

Answer

$$\tan^2\theta\cos^2\theta = \frac{\sin^2\theta}{\cos^2\theta}\times\cos^2\theta$$
$$=\sin^2\theta$$
$$=1-\cos^2\theta$$

Hence Proved.

4. Question

Prove the following trigonometric identities:

$$\cos ec\theta\sqrt{1-\cos^2\theta} = 1$$

Answer

$$\cos ec\theta\sqrt{1-\cos^2\theta} = \frac{1}{\sin\theta}\sqrt{\sin^2\theta}$$
$$=\frac{1}{\sin\theta}\times\sin\theta$$
$$=1$$

Hence Proved.

5. Question

Prove the following trigonometric identities:

$$(\sec^2\theta-1)(\cos ec^2\theta-1) = 1$$

Answer

$$(\sec^2\theta-1)(\cos ec^2\theta-1) = \tan^2\theta\times\cot^2\theta$$
$$=1$$

Hence Proved.

6. Question

Prove the following trigonometric identities:

$$\tan\theta+\frac{1}{\tan\theta} = \sec\theta\cos ec\theta$$

Answer

$$\tan\theta+\frac{1}{\tan\theta} = \frac{\tan^2\theta+1}{\tan\theta}$$
$$=\sec^2\theta\times\frac{\cos\theta}{\sin\theta}$$
$$=\frac{1}{\cos^2\theta}\times\frac{\cos\theta}{\sin\theta}$$
$$=\frac{1}{\cos\theta}\times\frac{1}{\sin\theta}$$
$$=\sec\theta\cos ec\theta$$

Hence Proved.

7. Question

Prove the following trigonometric identities:

$$\frac{\cos\theta}{1-\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$$

Answer

$$\frac{\cos\theta}{1-\sin\theta} = \frac{\cos\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}$$
$$=\frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$$
$$=\frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$$
$$=\frac{1+\sin\theta}{\cos\theta}$$

Hence Proved.

8. Question

Prove the following trigonometric identities:

$$\frac{\cos\theta}{1+\sin\theta}=\frac{1-\sin\theta}{\cos\theta}$$

Answer

$$\begin{aligned}\frac{\cos\theta}{1+\sin\theta}&=\frac{\cos\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}\\&=\frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}\\&=\frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}\\&=\frac{1-\sin\theta}{\cos\theta}\end{aligned}$$

Hence Proved.

9. Question

Prove the following trigonometric identities:

$$\cos^2 A+\frac{1}{1+\cot^2 A}=1$$

Answer

$$\begin{aligned}\cos^2 A+\frac{1}{1+\cot^2 A}&=\cos^2 A+\frac{1}{\operatorname{cosec}^2 A}\\&=\cos^2 A+\sin^2 A\\&=1\end{aligned}$$

Hence Proved.

10. Question

Prove the following trigonometric identities:

$$\sin^2 A+\frac{1}{1+\tan^2 A}=1$$

Answer

$$\begin{aligned}\sin^2 A+\frac{1}{1+\tan^2 A}&=\sin^2 A+\frac{1}{\sec^2 A}\\&=\sin^2 A+\cos^2 A\\&=1\end{aligned}$$

Hence Proved.

11. Question

Prove the following trigonometric identities:

$$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\operatorname{cosec}\theta-\cot\theta a$$

Answer

$$\begin{aligned}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}&=\sqrt{\frac{1-\cos\theta}{1+\cos\theta}\times\frac{1-\cos\theta}{1-\cos\theta}}\\&=\sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}\\&=\sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}\\&=\frac{1-\cos\theta}{\sin\theta}\\&=\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\\&=\operatorname{cosec}\theta-\cot\theta\end{aligned}$$

Hence Proved.

12. Question

Prove the following trigonometric identities:

$$\frac{1-\cos\theta}{\sin\theta}=\frac{\sin\theta}{1+\cos\theta}$$

Answer

$$\begin{aligned}\frac{1-\cos\theta}{\sin\theta}&=\frac{1-\cos\theta}{\sin\theta}\times\frac{1+\cos\theta}{1+\cos\theta}\\&=\frac{1-\cos^2\theta}{\sin\theta(1+\cos\theta)}\\&=\frac{\sin^2\theta}{\sin\theta(1+\cos\theta)}\\&=\frac{\sin\theta}{1+\cos\theta}\end{aligned}$$

Hence Proved.

13. Question

Prove the following trigonometric identities:

$$\frac{\sin\theta}{1-\cos\theta}=\operatorname{cosec}\theta+\cot\theta$$

Answer

$$\begin{aligned}\frac{\sin\theta}{1-\cos\theta}&=\frac{\sin\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}\\&=\frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}\\&=\frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}\\&=\frac{1+\cos\theta}{\sin\theta}\\&=\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}\\&=\operatorname{cosec}\theta+\cot\theta\end{aligned}$$

$$As,\ \frac{1}{\sin\ \theta}=\text{cosec}\ \theta$$

$$and\ \frac{\cos\ \theta}{\sin\ \theta}=\cot\ \theta$$

Hence Proved.

14. Question

Prove the following trigonometric identities:

$$\frac{1-\sin\theta}{1+\sin\theta}=(\sec\theta-\tan\theta)^2$$

Answer

$$\begin{aligned}\frac{1-\sin\theta}{1+\sin\theta}&=\frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}\\&=\frac{(1-\sin\theta)^2}{1-\sin^2\theta}\\&=\frac{(1-\sin\theta)^2}{\cos^2\theta}\\&=\left(\frac{1-\sin\theta}{\cos\theta}\right)^2\\&=\left(\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\right)^2\\&=(\sec\theta-\tan\theta)^2\end{aligned}$$

Hence Proved.

15. Question

Prove the following trigonometric identities:

$(\cos ec\theta+\sin\theta)(\cos ec\theta-\sin\theta)=\cot^2\theta+\cos^2\theta$

Answer

Consider,

(cosecθ + sinθ)(cosecθ - sinθ)

Apply the formula $(a^2 - b^2) = (a+b)(a-b)$

we get,

(cosecθ + sinθ)(cosecθ - sinθ) = $\text{cosec}^2\theta - \sin^2\theta$

As we know $1+\cot^2 A = \text{cosec}^2 A$

and $1-\cos^2 A = \sin^2 A$

So,

(cosecθ + sinθ)(cosecθ - sinθ) = $(1+\cot^2 A) -(1-\cos^2 A\ )$

$= 1+\cot^2 A - 1+\cos^2 A$

$=\cot^2 A +\cos^2 A$

Hence Proved.

16. Question

Prove the following trigonometric identities:

$\frac{(1+\cot^2\theta)\tan\theta}{\sec^2\theta}=\cot\theta$

Answer

To prove: $\frac{(1+\cot^2\theta)\tan\theta}{\sec^2\theta}=\cot\theta$

Proof: Use the identity $\text{cosec}^2\theta = 1 + \cot^2\theta$and the formula cosθ=1/ secθ and cosecθ=1/sinθ, tanθ= 1 / cotθcotθ = cosθ / sinθ

$$\frac{(1+\cot^2\theta)\tan\theta}{\sec^2\theta}=\frac{\cos ec^2\theta\times\tan\theta}{\sec^2\theta}$$
$$=\frac{\cos^2\theta\times\tan\theta}{\sin^2\theta}$$
$$=\frac{\cos^2\theta}{\sin^2\theta}\times\tan\theta$$
$$=\cot^2\theta\times\tan\theta$$
$$=\cot^2\theta\times\frac{1}{\cot\theta}$$
$$=\cot\theta$$

Hence Proved.

17. Question

Prove the following trigonometric identities:

$(\sec\theta+\cos\theta)(\sec\theta-\cos\theta)=\tan^2\theta+\sin^2\theta$

Answer

To Prove: $(\sec\theta+\cos\theta)(\sec\theta-\cos\theta)=\tan^2\theta+\sin^2\theta$

Proof: Use the formula:$(a + b) (a - b) = a^2 - b^2$ on (secθ + cosθ) (secθ - cosθ)

Where a = secθ and b = cosθ

so,

(secθ + cosθ) (secθ - cosθ) = $\sec^2\theta - \cos^2\theta$ ...... (1)

We know,$\sec^2\theta = \tan^2\theta + 1$

$\sin^2\theta + \cos^2\theta = 1$

Use the identities in the eq. (1)$(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) = \sec^2\theta - \cos^2\theta$

$= (\tan^2\theta + 1) - (1 - \sin^2\theta)$

$= \tan^2\theta + 1 - 1 + \sin^2\theta$

$= \tan^2\theta + \sin^2\theta$ Hence proved.

18. Question

Prove the following trigonometric identities:

$\sec A(1 - \sin A)(\sec A + \tan A) = 1$

Answer

$$\sec A(1-\sin A)(\sec A+\tan A)$$
$$= \left(\sec A - \frac{1}{\cos A} \times \sin A\right)(\sec A + \tan A)$$
$$= (\sec A - \tan A)(\sec A + \tan A)$$
$$= (\sec^2 A - \tan^2 A)$$
$$= (\tan^2 A + 1 - \tan^2 A)$$
$$= 1$$

Hence Proved.

19. Question

Prove the following trigonometric identities:

$(\operatorname{cosec} A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$

Answer

taking LHS

(cosecA-sinA)(secA-cosA)(tanA+cotA)As we know,cosec A = 1/sinAsec A = 1/cos Atan A = sin / cos ASo,

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= \left(\frac{1-\sin^2 A}{sinA}\right)\left(\frac{1-\cos^2 A}{cosA}\right)\left(\frac{sin^2 A + \cos^2 A}{cosA}\right)$$

As we know,$\sin^2$A+ $\cos^2$A = 1

$$= \left(\frac{\cos^2 A}{sinA}\right)\left(\frac{\sin^2 A}{cosA}\right)\left(\frac{1}{cosA}\right) = 1$$

Hence Proved.

20. Question

Prove the following trigonometric identities:

$\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$

Answer

$$
\begin{aligned}
LHS.: \tan^2\theta - \sin^2\theta &= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta \\
&= \frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta} \\
&= \frac{\sin^2\theta - \sin^2\theta(1-\sin^2\theta)}{\cos^2\theta} \\
&= \frac{\sin^2\theta - \sin^2\theta + \sin^4\theta}{\cos^2\theta} \\
&= \frac{\sin^2\theta}{\cos^2\theta} \times \sin^2\theta \\
&= \tan^2\theta \sin^2\theta \\
&= R.H.S
\end{aligned}
$$

Hence Proved.

21. Question

Prove the following trigonometric identities:

$(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta) = 1$

Answer

$$
\begin{aligned}
&(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta) \\
&= \sec^2\theta(1-\sin^2\theta) \\
&= \sec^2\theta\cos^2\theta \\
&= 1
\end{aligned}
$$

Hence Proved.

22. Question

Prove the following trigonometric identities:

$\sin^2 A\cot^2 A + \cos^2 A\tan^2 A = 1$

Answer

given : $\sin^2 A\cot^2 A + \cos^2 A\tan^2 A = 1$

To prove : Above equality holds.

Proof: Consider LHS,we know,

$$\cot\theta = \frac{\cos\theta}{\sin\theta} \ and \ \tan\theta = \frac{\sin\theta}{\cos\theta}$$

using these

$$
\begin{aligned}
&\sin^2 A\cot^2 A + \cos^2 A\tan^2 A \\
&= \sin^2 A \times \frac{\cos^2 A}{\sin^2 A} + \cos^2 A \times \frac{\sin^2 A}{\cos^2 A} \\
&= \cos^2 A + \sin^2 A \\
&= 1
\end{aligned}
$$

Which is equal to RHS.

Hence Proved.

23 A. Question

Prove the following trigonometric identities:

$\cot\theta - \tan\theta = \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$

Answer

$$L.H.S: \cot\theta - \tan\theta = \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$$
$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta.\cos\theta}$$
$$= \frac{\cos^2\theta - \left(1-\cos^2\theta\right)}{\sin\theta.\cos\theta}$$
$$= \frac{\cos^2\theta - 1 + \cos^2\theta}{\sin\theta.\cos\theta}$$
$$= \frac{2\cos^2\theta - 1}{\sin\theta.\cos\theta} = R.H.S$$

Hence Proved.

23 B. Question

Prove the following trigonometric identities:

$$\tan\theta - \cot\theta = \frac{2\sin^2\theta - 1}{\sin\theta\cos\theta}$$

Answer

$$L.H.S: \quad \tan\theta - \cot\theta = \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{\sin^2\theta - \left(1-\sin^2\theta\right)}{\sin\theta\cos\theta}$$
$$= \frac{\sin^2\theta - 1 + \sin^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{2\sin^2\theta - 1}{\sin\theta\cos\theta}$$

Hence Proved.

24. Question

Prove the following trigonometric identities:

$$\frac{\cos^2\theta}{\sin\theta} - \cos ec\theta + \sin\theta = 0$$

Answer

$$\frac{\cos^2\theta}{\sin\theta} - \cos ec\theta + \sin\theta = \frac{\cos^2\theta}{\sin\theta} - \frac{1}{\sin\theta} + \sin\theta$$
$$= \frac{\cos^2\theta - 1 + \sin^2\theta}{\sin\theta}$$
$$= \frac{\left(\cos^2\theta + \sin^2\theta\right) - 1}{\sin\theta}$$
$$= \frac{1-1}{\sin\theta} = 0$$

Hence Proved.

25. Question

Prove the following trigonometric identities:

$$\frac{1}{1+\sin A} + \frac{1}{1-\sin A} = 2\sec^2 A$$

Answer

$$\frac{1}{1+\sin A} + \frac{1}{1-\sin A} = \frac{1-\sin A + 1 + \sin A}{1-\sin^2 A}$$
$$= \frac{2}{\cos^2 A}$$
$$= 2\sec^2 A$$

Hence Proved.

26. Question

Prove the following trigonometric identities:

$$\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}=2\sec\theta$$

Answer

$$=\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}=\frac{1+\sin^2\theta+2\sin\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$=\frac{1+1+2\sin\theta}{\cos\theta(1+\sin\theta)}$$

$$=\frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)}$$

$$=\frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$$

$$=\frac{2}{\cos\theta}$$

$$=2\sec\theta$$

Hence Proved.

27. Question

Prove the following trigonometric identities:

$$\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta}=\frac{1+\sin^2\theta}{1-\sin^2\theta}$$

Answer

$$\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta}=\frac{1+\sin^2\theta}{1-\sin^2\theta}$$

$$=\frac{1+\sin^2\theta+2\sin\theta+1+\sin^2\theta-2\sin\theta}{2\cos^2\theta}$$

$$=\frac{2+2\sin^2\theta}{2\cos^2\theta}$$

$$=\frac{2(1+\sin^2\theta)}{2\cos^2\theta}$$

$$=\frac{(1+\sin^2\theta)}{\cos^2\theta}$$

$$=\frac{1+\sin^2\theta}{1-\sin^2\theta}$$

Hence Proved.

28. Question

Prove the following trigonometric identities:

$$\frac{1+\tan^2\theta}{1+\cot^2\theta}=\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2=\tan^2\theta$$

Answer

Use the formula:

$$\tan\theta=\frac{\sin\theta}{\cos\theta}\ and\ \cot\theta=\frac{\cos\theta}{\sin\theta}$$

$$\frac{1+\tan^2\theta}{1+\cot^2\theta}=\frac{1+\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\cos^2\theta}{\sin^2\theta}}=\frac{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}}$$

$$=\frac{\sin^2\theta}{\cos^2\theta}=\tan^2\theta$$

Now,

$$\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2=\left(\frac{1-\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}\right)=\left(\frac{\frac{\cos\theta-\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}\right)^2$$

$$=\left(\frac{\frac{\cos\theta-\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}\right)^2=\left(\frac{(\cos\theta-\sin\theta)}{\cos\theta}\times\frac{\sin\theta}{-(\cos\theta-\sin\theta)}\right)^2$$

$$=\left(\frac{\sin\theta}{\cos\theta}\right)^2$$

$$=\tan^2\theta$$

Hence Proved.

29. Question

Prove the following trigonometric identities:

$$\frac{1+\sec\theta}{\sec\theta}=\frac{\sin^2\theta}{1-\cos\theta}$$

Answer

$$\frac{1+\sec\theta}{\sec\theta}=\frac{1+\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$$=\frac{1+\cos\theta}{1}$$

$$=\frac{(1+\cos\theta)(1-\cos\theta)}{(1-\cos\theta)}$$

$$=\frac{1-\cos^2\theta}{(1-\cos\theta)}$$

$$=\frac{\sin^2\theta}{1-\cos\theta}$$

Hence Proved.

30. Question

Prove the following trigonometric identities:

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\tan\theta+\cot\theta$$

Answer

Given : $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\tan\theta+\cot\theta$

To prove: Above equality.

Taking LHS Use $\tan\theta=\frac{1}{\cot\theta}$, $cot\theta=\frac{1}{tan\theta}$

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$$

$$=\frac{\tan\theta}{1-\frac{1}{\tan\theta}}+\frac{\frac{1}{tan\,\theta}}{1-\tan\theta}$$

$$=\frac{\tan^2\theta}{\tan\theta-1}+\frac{1}{\tan\theta\,(1-\tan\theta)}$$

$$=\frac{1}{(\tan\theta-1)}\left(\tan^2\theta-\frac{1}{\tan\theta}\right)$$

$$=\frac{\tan^3\theta-1}{\tan\theta\,(\tan\theta-1)}$$

$$=\frac{(\tan\theta-1)(\tan^2\theta+\tan\theta+1)}{\tan\theta\,(\tan\theta-1)}$$

[using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

$$=\frac{\tan^2\theta+\tan\ \theta+1}{\tan\ \theta}$$

$$=\tan\ \theta+1+\cot\ \theta$$

= RHS**Hence Proved.**

31. Question

Prove the following trigonometric identities:

$\sec^6\theta=\tan^6\theta+3\tan^2\theta\sec^2\theta+1$

Answer

Taking RHS

$\tan^6\theta + 3\tan^2\theta\sec^2\theta + 1$

$= (\sec^2\theta - 1)^3 + 3(\sec^2\theta - 1)\sec^2\theta + 1$

[As, $\tan^2\theta = \sec^2\theta - 1$]

$= (\sec^6\theta - 1 - 3\sec^4\theta + 3\sec^2\theta) + (3\sec^4\theta - 3\sec^2\theta) + 1$

[$(a + b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$]

$= \sec^6\theta$= LHSHence Proved.

32. Question

Prove the following trigonometric identities:

$\text{cosec}^6\theta=\cot^6\theta+3\cot^2\theta\,\text{cosec}^2\theta+1$

Answer

$\text{cosec}^6\theta = \cot^6\theta + 3\cot^2\theta\,\text{cosec}^2\theta + 1$

$\text{cosec}^6\theta - \cot^6\theta - 3\cot^2\theta\,\text{cosec}^2\theta = 1 \quad ...(i)$

since we know that

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

so we can write LHS of eq. (i) as

$(\text{cosec}^2\theta)^3 - (\cot^2\theta)^3 - 3\cot^2\theta\,\text{cosec}^2\theta\,(\text{cosec}^2\theta - \cot^2\theta) \quad ......(\because \text{cosec}^2\theta - \cot^2\theta = 1)$

$= (\text{cosec}^2\theta - \cot^2\theta)^3$

$= 1 = RHS$

Hence Proved

33. Question

Prove the following trigonometric identities:

$$\frac{(1+\tan^2\theta)\cot\theta}{\text{cosec}^2\theta} = \tan\theta$$

Answer

$$\frac{(1+\tan^2\theta)\cot\theta}{\text{cosec}^2\theta} = \frac{\sec^2\theta \times \cot\theta}{\text{cosec}^2\theta}$$
$$= \frac{\sin^2\theta}{\cos^2\theta} \times \cot\theta$$
$$= \tan^2\theta \times \cot\theta$$
$$= \tan^2\theta \times \frac{1}{\tan\theta}$$
$$= \tan\theta$$

Hence Proved.

34. Question

Prove the following trigonometric identities:

$$\frac{1+\cos A}{\sin^2 A} = \frac{1}{1-\cos A}$$

Answer

$$\frac{1+\cos A}{\sin^2 A} = \frac{1+\cos A}{\sin^2 A} \times \frac{1-\cos A}{1-\cos A}$$
$$= \frac{1-\cos^2 A}{\sin^2 A(1-\cos A)}$$
$$= \frac{\sin^2 A}{\sin^2 A(1-\cos A)}$$
$$= \frac{1}{1-\cos A}$$

Hence Proved.

35. Question

Prove the following trigonometric identities:

$$\frac{\sec A - \tan A}{\sec A + \tan A} = \frac{\cos^2 A}{(1+\sin A)^2}$$

Answer

$$R.H.S: \frac{\cos^2 A}{(1+\sin A)^2} = \frac{1-\sin^2 A}{(1+\sin A)^2}$$
$$= \frac{(1-\sin A)(1+\sin A)}{(1+\sin A)^2}$$
$$= \frac{(1-\sin A)/\cos A}{(1+\sin A)/\cos A}$$
$$= \frac{\left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right)}{\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)}$$
$$= \frac{\sec A - \tan A}{\sec A + \tan A} = L.H.S$$

Hence Proved.

36. Question

Prove the following trigonometric identities:

$$\frac{1+\cos A}{\sin A}=\frac{\sin A}{1-\cos A}$$

Answer

$$\begin{aligned}\frac{1+\cos A}{\sin A}&=\frac{1+\cos A}{\sin A}\times\frac{1-\cos A}{1-\cos A}\\&=\frac{1-\cos^2 A}{\sin A(1-\cos A)}\\&=\frac{\sin^2 A}{\sin A(1-\cos A)}\\&=\frac{\sin A}{1-\cos A}\end{aligned}$$

Hence Proved.

37. Question

Prove the following trigonometric identities:

$$\sqrt{\frac{1+\sin A}{1-\sin A}}=\sec A+\tan A$$

Answer

$$\begin{aligned}\sqrt{\frac{1+\sin A}{1-\sin A}}&=\sqrt{\frac{1+\sin A}{1-\sin A}\times\frac{1+\sin A}{1+\sin A}}\\&=\sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}\\&=\sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}\\&=\frac{1+\sin A}{\cos A}\\&=\frac{1}{\cos A}+\frac{\sin A}{\cos A}\\&=\sec A+\tan A\end{aligned}$$

Hence Proved.

38. Question

Prove the following trigonometric identities:

$$\sqrt{\frac{1-\cos A}{1+\cos A}}+\sqrt{\frac{1+\cos A}{1-\cos A}}=2\operatorname{cosec} A$$

Answer

$$\begin{aligned}\sqrt{\frac{1-\cos A}{1+\cos A}}+\sqrt{\frac{1+\cos A}{1-\cos A}}&=\sqrt{\frac{1-\cos A}{1+\cos A}\times\frac{1-\cos A}{1-\cos A}}+\sqrt{\frac{1+\cos A}{1+\cos A}\times\frac{1+\cos A}{1+\cos A}}\\&=\sqrt{\frac{(1-\cos A)^2}{1-\cos^2 A}}+\sqrt{\frac{(1+\cos A)^2}{1-\cos^2 A}}\\&=\sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}+\sqrt{\frac{(1+\cos A)^2}{\sin^2 A}}\\&=\frac{(1-\cos A)}{\sin A}+\frac{(1+\cos A)}{\sin A}\\&=\frac{1}{\sin A}-\frac{\cos A}{\sin A}+\frac{1}{\sin A}+\frac{\cos A}{\sin A}\\&=\frac{2}{\sin A}\\&=2\operatorname{cosec} A\end{aligned}$$

Hence Proved.

39. Question

Prove the following trigonometric identities:

$$(\sec A-\tan A)^2=\frac{1-\sin A}{1+\sin A}$$

Answer

$$(\sec A - \tan A)^2 = \left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right)^2$$
$$= \frac{(1-\sin A)^2}{\cos^2 A}$$
$$= \frac{(1-\sin A)^2}{1-\sin^2 A}$$
$$= \frac{(1-\sin A)^2}{(1-\sin A)(1+\sin A)}$$
$$= \frac{1-\sin A}{1+\sin A}$$

Hence Proved.

40. Question

Prove the following trigonometric identities:

$$\frac{1-\cos A}{1+\cos A} = (\cot A - \cos ecA)^2$$

Answer

Given: $\frac{1-\cos A}{1+\cos A} = (\cot A - \cos ecA)^2$

To prove: Above equality

Proof: Rationalize the LHS,Use $\sin^2 x + \cos^2 x = 1$Solve,

$$\frac{1-\cos A}{1+\cos A} = \frac{1-\cos A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}$$
$$= \frac{(1-\cos A)^2}{1-\cos^2 A}$$
$$= \frac{(1-\cos A)^2}{\sin^2 A}$$
$$= \left(\frac{1-\cos A}{\sin A}\right)^2$$
$$= \left(\frac{1}{\sin A} - \frac{\cos A}{\sin A}\right)^2$$
$$= (\cot A - \cos ecA)^2$$

Hence proved

41. Question

Prove the following trigonometric identities:

$$\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} = 2\cos ecA \cot A$$

Answer

$$\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} = \frac{\sec A + 1 + \sec A - 1}{\sec^2 A - 1}$$
$$= \frac{2\sec A}{\tan^2 A}$$
$$= \frac{2}{\cos A} \times \frac{\cos A}{\sin^2 A}$$
$$= 2\cos ecA \cot A$$

Hence Proved.

42. Question

Prove the following trigonometric identities:

$$\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A$$

Answer

$$\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}=\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}$$
$$=\frac{\cos A}{1-\frac{\sin A}{\cos A}}+\frac{\sin A}{1-\frac{\cos A}{\sin A}}$$
$$=\frac{\cos A}{\frac{\cos A-\sin A}{\cos A}}+\frac{\sin A}{\frac{\sin A-\cos A}{\sin A}}$$
$$=\frac{\cos^2 A}{\cos A-\sin A}-\frac{\sin^2 A}{\cos A-\sin A}$$
$$=\frac{\cos^2 A-\sin^2 A}{\cos A-\sin A}$$
$$=\frac{(\cos A-\sin A)(\cos A+\sin A)}{\cos A-\sin A}$$
$$=\sin A+\cos A$$

Hence Proved.

43. Question

Prove the following trigonometric identities:

$$\frac{\operatorname{cosec} A}{\operatorname{cosec} A-1}+\frac{\operatorname{cosec} A}{\operatorname{cosec} A+1}=2\sec^2 A$$

Answer

$$\frac{\operatorname{cosec} A}{\operatorname{cosec} A-1}+\frac{\operatorname{cosec} A}{\operatorname{cosec} A+1}=\frac{\operatorname{cosec} A(\operatorname{cosec} A+1)+\operatorname{cosec} A(\operatorname{cosec} A-1)}{\operatorname{cosec}^2 A-1}$$
$$=\frac{\operatorname{cosec}^2 A+\operatorname{cosec} A+\operatorname{cosec}^2 A-\operatorname{cosec} A}{\operatorname{cosec}^2 A-1}$$
$$=\frac{2\operatorname{cosec}^2 A}{\cot^2 A}=\frac{2}{\sin^2 A}\times\frac{\sin^2 A}{\cos^2 A}=2\sec^2 A$$

Hence Proved.

44. Question

Prove the following trigonometric identities:

$$(1+\tan^2 A)+\left(1+\frac{1}{\tan^2 A}\right)=\frac{1}{\sin^2 A-\sin^4 A}$$

Answer

$$(1+\tan^2 A)+\left(1+\frac{1}{\tan^2 A}\right)=\sec^2 A+\frac{1+\tan^2 A}{\tan^2 A}$$
$$=\sec^2 A+\frac{\sec^2 A}{\tan^2 A}$$
$$=\frac{1}{\cos^2 A}+\frac{1}{\cos^2 A}\times\frac{\cos^2 A}{\sin^2 A}$$
$$=\frac{1}{\cos^2 A}+\frac{1}{\sin^2 A}$$
$$=\frac{1}{1-\sin^2 A}+\frac{1}{\sin^2 A}$$
$$=\frac{\sin^2 A+1-\sin^2 A}{(1-\sin^2 A)\sin^2 A}$$
$$=\frac{1}{\sin^2 A-\sin^4 A}$$

Hence Proved.

45. Question

Prove the following trigonometric identities:

$$\frac{\tan^2 A}{1+\tan^2 A}+\frac{\cot^2 A}{1+\cot^2 A}=1$$

Answer

$$\frac{\tan^2 A}{1+\tan^2 A}+\frac{\cot^2 A}{1+\cot^2 A}=\frac{\tan^2 A}{\sec^2 A}+\frac{\cot^2 A}{\operatorname{cosec}^2 A}$$
$$=\frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{1}{\cos^2 A}}+\frac{\frac{\cos^2 A}{\sin^2 A}}{\frac{1}{\sin^2 A}}$$
$$=\sin^2 A+\cos^2 A$$
$$=1$$

Hence Proved.

46. Question

Prove the following trigonometric identities:

$$\frac{\cot A-\cos A}{\cot A+\cos A}=\frac{\operatorname{cosec} A-1}{\operatorname{cosec} A+1}$$

Answer

$$\frac{\cot A-\cos A}{\cot A+\cos A}=\frac{\frac{\cos A}{\sin A}-\cos A}{\frac{\cos A}{\sin A}+\cos A}$$
$$=\frac{\cos A\left(\frac{1}{\sin A}-1\right)}{\cos A\left(\frac{1}{\sin A}+1\right)}$$
$$=\frac{\operatorname{cosec} A-1}{\operatorname{cosec} A+1}$$

Hence Proved.

47 A. Question

Prove the following trigonometric identities:

$$\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}=\frac{1+\sin\theta}{\cos\theta}$$

Answer

$$\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}=\frac{\{(1+\cos\theta)+\sin\theta\}}{\{(1+\cos\theta)-\sin\theta\}}\times\frac{\{(1+\cos\theta)+\sin\theta\}}{\{(1+\cos\theta)+\sin\theta\}}$$
$$=\frac{\{(1+\cos\theta)+\sin\theta\}^2}{(1+\cos\theta)^2-\sin^2\theta}$$
$$=\frac{(1+\cos\theta)^2+\sin^2\theta+2(1+\cos\theta)\sin\theta}{(1+\cos\theta)^2-\sin^2\theta}$$
$$=\frac{1+\cos^2\theta+2\cos\theta+\sin^2\theta+2\sin\theta+2\sin\theta\cos\theta}{1+\cos^2\theta+2\cos\theta-\sin^2\theta}$$
$$=\frac{1+\cos^2\theta+\sin^2\theta+2\cos\theta+2\sin\theta+2\sin\theta\cos\theta}{1+\cos^2\theta+2\cos\theta-1+\cos^2\theta}$$
$$=\frac{1+1+2\sin\theta+2\cos\theta(1+\sin\theta)}{2\cos^2\theta+2\cos\theta}$$
$$=\frac{2(1+\sin\theta)+2\cos\theta(1+\sin\theta)}{2\cos\theta(1+\cos\theta)}$$
$$=\frac{2(1+\sin\theta)(1+\cos\theta)}{2\cos\theta(1+\cos\theta)}$$
$$=\frac{1+\sin\theta}{\cos\theta}$$

Hence Proved.

47 B. Question

Prove the following trigonometric identities:

$$\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{1}{\sec\theta-\tan\theta}$$

Answer

To Prove: $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{1}{\sec\theta-\tan\theta}$

$$\text{L.H.S} = \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$$

Dividing the numerator and denominator by cosθ, we get,

$$= \frac{\frac{\sin\theta - \cos\theta + 1}{\cos\theta}}{\frac{\sin\theta + \cos\theta - 1}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}}$$

$$= \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$$

$$= \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1}$$

Now, we know that, $\sec^2\theta - \tan^2\theta = 1$

Therefore, replacing 1 by $\sec^2\theta - \tan^2\theta$ in the numerator only, we get,

$$= \frac{(\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$$

As we know, $a^2 - b^2 = (a-b)(a+b)$

$$= \frac{(\tan\theta + \sec\theta) - [(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)]}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan\theta + \sec\theta)[1 - (\sec\theta - \tan\theta)]}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan\theta + \sec\theta)[1 - \sec\theta + \tan\theta]}{\tan\theta - \sec\theta + 1}$$

= secθ + tanθNow, multiplying and dividing by secθ - tanθ, we get,

$$= \frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} \times (\sec\theta - \tan\theta)$$

$$= \frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta}$$

As we know, $\sec^2\theta - \tan^2\theta = 1$

$$= \frac{1}{\sec\theta - \tan\theta}$$

= R.H.SHence, proved.

47 C. Question

Prove the following trigonometric identities:

$$\frac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} = \cos ec\theta + \cot\theta$$

Answer

$$\text{L.H.S} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} + \frac{1}{\sin A}}$$

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{\{(\cot A) - (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}{\{(\cot A) + (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}$$

$$= \frac{(\cot A - 1 + \operatorname{cosec} A)^2}{(\cot A)^2 - (1 - \operatorname{cosec} A)^2}$$

$$= \frac{\cot^2 A + 1 + \operatorname{cosec}^2 A - 2\cot A - 2\operatorname{cosec} A + 2\cot A \operatorname{cosec} A}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A)}$$

$$= \frac{2\operatorname{cosec}^2 A + 2\cot A \operatorname{cosec} A - 2\cot A - 2\operatorname{cosec} A}{\cot^2 A - 1 - \operatorname{cosec}^2 A + 2\operatorname{cosec} A}$$

$$= \frac{2\operatorname{cosec} A(\operatorname{cosec} A + \cot A) - 2(\cot A + \operatorname{cosec} A)}{\cot^2 A - \operatorname{cosec}^2 A - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{-1 - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{(2\operatorname{cosec} A - 2)}$$

$= \operatorname{cosec} A + \cot A$

Hence Proved.

48. Question

Prove the following trigonometric identities:

$$\frac{1}{\sec A + \tan A} - \frac{1}{\cos A} = \frac{1}{\cos A} - \frac{1}{\sec A - \tan A}$$

Answer

To prove: $\frac{1}{\sec A + \tan A} - \frac{1}{\cos A} = \frac{1}{\cos A} - \frac{1}{\sec A - \tan A}$

Proof: Consider LHS,

$$\frac{1}{\sec A + \tan A} - \frac{1}{\cos A}$$

Use the formula:secθ = 1/cos θ and tanθ=sinθ/cosθ

$$= \frac{1}{\frac{1}{\cos A} + \frac{\sin A}{\cos A}} - \frac{1}{cos A}$$

$$= \frac{\cos A}{1 + \sin A} - \frac{1}{cos A}$$

Do rationalization,

$$= \left(\frac{cos A}{1 + sin A} \times \frac{1 - \sin A}{1 - \sin A}\right) - \frac{1}{cos A}$$

$$= \frac{cosA(1-sinA)}{(1+sinA)(1-sinA)} - \frac{1}{cosA}$$

$$= \frac{cosA(1-sinA)}{\left(1-sin^2A\right)} - \frac{1}{cosA}$$

Use the formula $\cos^2\theta + \sin^2\theta = 1$

$$= \frac{\cos A(1-\sin A)}{\cos^2 A} - \frac{1}{\cos A}$$

$$= \frac{(1-sinA)}{cos\ \ A} - \frac{1}{cosA}$$

$$= \frac{1}{\cos A} - \frac{\sin A}{\cos A} - \frac{1}{\cos A}$$

= - tan AConsider RHS, $\frac{1}{\cos A} - \frac{1}{\sec A - \tan A}$

Use the formula:secθ = 1/cos θ and tanθ=sinθ/cosθ

$$= \frac{1}{cosA} - \frac{1}{\frac{1}{\cos A} - \frac{\sin A}{\cos A}}$$

$$= \frac{1}{cosA} - \frac{\cos A}{1-\sin A}$$

Do rationalization,

$$= \frac{1}{cosA} - \left(\frac{cosA}{1-sinA} \times \frac{1+\sin A}{1+\sin A}\right)$$

$$= \frac{1}{cosA} - \frac{cosA(1+sinA)}{(1-sinA)(1+sinA)}$$

$$= \frac{1}{cosA} - \frac{cosA(1+\sin A)}{\left(1-sin^2A\right)}$$

Use the formula $\cos^2\theta + \sin^2\theta = 1$

$$= \frac{1}{\cos A} - \frac{\cos A(1+\sin A)}{\cos^2 A}$$

$$= \frac{1}{cosA} - \frac{(1+sinA)}{cosA}$$

$$= \frac{1}{cosA} - \frac{1}{\cos A} - \frac{\sin A}{\cos A}$$

= - tan ALHS = RHSHence proved.

49. Question

Prove the following trigonometric identities:

$\tan^2 A + \cot^2 A = \sec^2 A \operatorname{cosec}^2 A - 2$

Answer

$$\begin{aligned} RHS &= \sec^2 A \operatorname{cosec}^2 A - 2 \\ &= (1+\tan^2 A)(1+\cot^2 A) - 2 \\ &= (\tan^2 A + \cot^2 A + \tan^2 A \cot^2 A + 1) - 2 \\ &= (\tan^2 A + \cot^2 A + 1 + 1) - 2 \\ &= \tan^2 A + \cot^2 A \\ &= LHS \end{aligned}$$

Hence Proved.

50. Question

Prove the following trigonometric identities:

$\frac{1-\tan^2 A}{\cot^2 A - 1} = \tan^2 A$

Answer

To prove:

$\frac{1-\tan^2 A}{\cot^2 A - 1} = \tan^2 A$

Use the formula

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \ and \ \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\begin{aligned} \frac{1-\tan^2 A}{\cot^2 A - 1} &= \frac{1-\frac{\sin^2 A}{\cos^2 A}}{\frac{\cos^2 A}{\sin^2 A} - 1} = \\ &= \frac{\frac{\cos^2 A - \sin^2 A}{\cos^2 A}}{\frac{\cos^2 A - \sin^2 A}{\sin^2 A}} \\ &= \frac{\sin^2 A}{\cos^2 A} \\ &= \tan^2 A \end{aligned}$$

Hence Proved.

51. Question

Prove the following trigonometric identities:

$1 + \frac{\cot^2\theta}{1+\operatorname{cosec}\theta} = \operatorname{cosec}\theta$

Answer

$$\begin{aligned} 1 + \frac{\cot^2\theta}{1+\operatorname{cosec}\theta} &= 1 + \frac{\operatorname{cosec}^2\theta - 1}{1+\operatorname{cosec}\theta} \\ &= 1 + \frac{(\operatorname{cosec}\theta - 1)(\operatorname{cosec}\theta + 1)}{(1+\operatorname{cosec}\theta)} \\ &= 1 + \operatorname{cosec}\theta - 1 \\ &= \operatorname{cosec}\theta \end{aligned}$$

Hence Proved.

52. Question

Prove the following trigonometric identities:

$\frac{\cos\theta}{\operatorname{cosec}\theta + 1} + \frac{\cos\theta}{\operatorname{cosec}\theta - 1} = 2\tan\theta$

Answer

$$\frac{\cos\theta}{\operatorname{cosec}\theta+1}+\frac{\cos\theta}{\operatorname{cosec}\theta-1}=\frac{\cos\theta(\operatorname{cosec}\theta-1)+\cos\theta(\operatorname{cosec}\theta+1)}{(\operatorname{cosec}\theta+1)(\operatorname{cosec}\theta-1)}$$
$$=\frac{2\cot\theta}{\cot^2\theta}$$
$$=2\tan\theta$$

Hence Proved.

53. Question

Prove the following trigonometric identities:

$$\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}=\cot\theta$$

Answer

$$\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}=\frac{(1+\cos\theta)-(1-\cos^2\theta)}{\sin\theta(1+\cos\theta)}$$
$$=\frac{(1+\cos\theta)-(1+\cos\theta)(1-\cos\theta)}{\sin\theta(1+\cos\theta)}$$
$$=\frac{(1+\cos\theta)\{1-(1-\cos\theta)\}}{\sin\theta(1+\cos\theta)}$$
$$=\frac{\cos\theta}{\sin\theta}$$
$$=\cot\theta$$

Hence Proved.

54. Question

Prove the following trigonometric identities:

$$\frac{\tan^3\theta}{1+\tan^2\theta}+\frac{\cot^3\theta}{1+\cot^2\theta}=\sec\theta\operatorname{cosec}\theta-2\sin\theta\cos\theta$$

Answer

$$\frac{\tan^3\theta}{1+\tan^2\theta}+\frac{\cot^3\theta}{1+\cot^2\theta}=\frac{\tan^3\theta}{\sec^2\theta}+\frac{\cot^3\theta}{\operatorname{cosec}^2\theta}$$
$$=\tan^3\theta\cos^2\theta+\cot^3\theta\sin^2\theta$$
$$=\frac{\sin^3\theta}{\cos^3\theta}\cos^2\theta+\frac{\cos^3\theta}{\sin^3\theta}\sin^2\theta$$
$$=\frac{\sin^3\theta}{\cos\theta}+\frac{\cos^3\theta}{\sin\theta}$$
$$=\frac{\sin^4\theta+\cos^4\theta}{\sin\theta\cos\theta}$$
$$=\frac{1-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$
$$=\frac{1}{\sin\theta\cos\theta}-2\sin\theta\cos\theta$$
$$=\sec\theta\operatorname{cosec}\theta-2\sin\theta\cos\theta$$

Hence Proved.

55. Question

If $T_n=\sin^n\theta+\cos^n\theta$, prove that $\frac{T_3-T_5}{T_1}=\frac{T_5-T_7}{T_3}$

Answer

$T_n = \sin^n\theta + \cos^n\theta$

$T_1 = \sin^1\theta + \cos^1\theta = \sin\theta + \cos\theta$

$T_3 = \sin^3\theta + \cos^3\theta,\ T_5 = \sin^5\theta + \cos^5\theta$ and $T_7 = \sin^7\theta + \cos^7\theta$

Now, we have,

$$\frac{T_3 - T_5}{T_1} = \frac{\sin^3\theta + \cos^3\theta - (\sin^5\theta + \cos^5\theta)}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^3\theta + \cos^3\theta - \sin^5\theta - \cos^5\theta}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^3\theta(1 - \sin^2\theta) + \cos^3\theta(1 - \cos^2\theta)}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^3\theta\cos^2\theta + \cos^3\theta\sin^2\theta}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^2\theta\cos^2\theta(\sin\theta + \cos\theta)}{\sin\theta + \cos\theta}$$

$$= \sin^2\theta\cos^2\theta \quad \text{.... (1)}$$

$$\frac{T_5 - T_7}{T_3} = \frac{\sin^5\theta + \cos^5\theta - (\sin^7\theta + \cos^7\theta)}{\sin^3\theta + \cos^3\theta}$$

$$= \frac{\sin^5\theta + \cos^5\theta - \sin^7\theta - \cos^7\theta}{\sin^3\theta + \cos^3\theta}$$

$$= \frac{\sin^5\theta(1 - \sin^2\theta) + \cos^5\theta(1 - \cos^2\theta)}{\sin^3\theta + \cos^3\theta}$$

$$= \frac{\sin^5\theta\cos^2\theta + \cos^5\theta\sin^2\theta}{\sin^3\theta + \cos^3\theta}$$

$$= \frac{\sin^2\theta\cos^2\theta(\sin^3\theta + \cos^3\theta)}{\sin^3\theta + \cos^3\theta}$$

$$= \sin^2\theta\cos^2\theta \quad \text{.... (2)}$$

From(1) and (2), we get

$$\frac{T_3 - T_5}{T_1} = \frac{T_5 - T_7}{T_3}$$

Hence Proved.

56. Question

Prove the following trigonometric identities:

$$\left(\tan\theta + \frac{1}{\cos\theta}\right)^2 + \left(\tan\theta - \frac{1}{\cos\theta}\right)^2 = 2\left(\frac{1 + \sin^2\theta}{1 - \sin^2\theta}\right)$$

Answer

$$\left(\tan\theta + \frac{1}{\cos\theta}\right)^2 + \left(\tan\theta - \frac{1}{\cos\theta}\right)^2$$

$$= (\tan\theta + \sec\theta)^2 + (\tan\theta - \sec\theta)^2$$

$$= (\tan^2\theta + \sec^2\theta + 2\tan\theta\sec\theta) + (\tan^2\theta + \sec^2\theta - 2\tan\theta\sec\theta)$$

$$= \tan^2\theta + \sec^2\theta + 2\tan\theta\sec\theta + \tan^2\theta + \sec^2\theta - 2\tan\theta\sec\theta$$

$$= 2(\tan^2\theta + \sec^2\theta)$$

$$= 2\left(\frac{\sin^2\theta}{\cos^2\theta} + \frac{1}{\cos^2\theta}\right)$$

$$= 2\left(\frac{1 + \sin^2\theta}{\cos^2\theta}\right) = 2\left(\frac{1 + \sin^2\theta}{1 - \sin^2\theta}\right)$$

Hence Proved.

57. Question

Prove the following trigonometric identities:

$$\left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{co\sec^2\theta - \sin^2\theta}\right)\sin^2\theta\cos^2\theta = \frac{1 - \sin^2\theta\cos^2\theta}{2 + \sin^2\theta\cos^2\theta}$$

Answer

$$\left(\frac{1}{\sec^2\theta-\cos^2\theta}+\frac{1}{co\sec^2\theta-\sin^2\theta}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{1}{\frac{1}{\cos^2\theta}-\cos^2\theta}+\frac{1}{\frac{1}{\sin^2\theta}-\sin^2\theta}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{1}{\frac{1-\cos^4\theta}{\cos^2\theta}}+\frac{1}{\frac{1-\sin^4\theta}{\sin^2\theta}}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{\cos^2\theta}{1-\cos^4\theta}+\frac{\sin^2\theta}{1-\sin^4\theta}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{\cos^2\theta}{\cos^2\theta+\sin^2\theta-\cos^4\theta}+\frac{\sin^2\theta}{\cos^2\theta+\sin^2\theta-\sin^4\theta}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{\cos^2\theta}{\cos^2\theta(1-\cos^2\theta)+\sin^2\theta}+\frac{\sin^2\theta}{\cos^2\theta+\sin^2\theta(1-\sin^2\theta)}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{\cos^2\theta}{\cos^2\theta\sin^2\theta+\sin^2\theta}+\frac{\sin^2\theta}{\cos^2\theta+\sin^2\theta\cos^2\theta}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{\cos^2\theta}{\sin^2\theta(\cos^2\theta+1)}+\frac{\sin^2\theta}{\cos^2\theta(1+\sin^2\theta)}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{\cos^4\theta(1+\sin^2\theta)+\sin^4\theta(\cos^2\theta+1)}{\sin^2\theta\cos^2\theta(\cos^2\theta+1)(1+\sin^2\theta)}\right)\sin^2\theta\cos^2\theta$$

$$=\frac{\cos^4\theta(1+\sin^2\theta)+\sin^4\theta(\cos^2\theta+1)}{(\cos^2\theta+1)(1+\sin^2\theta)}$$

$$=\frac{(\cos^4\theta+\sin^2\theta\cos^4\theta)+(\sin^4\theta\cos^2\theta+\sin^4\theta)}{1+\cos^2\theta+\sin^2\theta+\cos^2\theta\sin^2\theta}$$

$$=\frac{(\cos^4\theta+\sin^4\theta)+\sin^2\theta\cos^2\theta(\cos^2\theta+\sin^2\theta)}{1+1+\cos^2\theta\sin^2\theta}$$

$$=\frac{1-2\sin^2\theta\cos^2\theta+\sin^2\theta\cos^2\theta\times 1}{2+\cos^2\theta\sin^2\theta}$$

$$=\frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$$

Hence Proved.

58. Question

Prove the following trigonometric identities:

$$\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2=\frac{1-\cos\theta}{1+\cos\theta}$$

Answer

$$\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2=\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\times\frac{1+\sin\theta-\cos\theta}{1+\sin\theta-\cos\theta}\right)^2$$
$$=\left[\frac{(1+\sin\theta-\cos\theta)^2}{(1+\sin\theta)^2-\cos^2\theta}\right]^2$$
$$=\left[\frac{1+\sin^2\theta+\cos^2\theta+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{1+\sin^2\theta+2\sin\theta-\cos^2\theta}\right]^2$$
$$=\left[\frac{1+1+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{1-\cos^2\theta+\sin^2\theta+2\sin\theta}\right]^2$$
$$=\left[\frac{2+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{\sin^2\theta+\sin^2\theta+2\sin\theta}\right]^2$$
$$=\left[\frac{2(1+\sin\theta)-2\cos\theta(1+\sin\theta)}{2\sin^2\theta+2\sin\theta}\right]^2$$
$$=\left[\frac{2(1+\sin\theta)(1-\cos\theta)}{2\sin\theta(1+\sin\theta)}\right]^2$$
$$=\frac{(1-\cos\theta)^2}{\sin^2\theta}$$
$$=\frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$
$$=\frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}$$
$$=\frac{1-\cos\theta}{1+\cos\theta}$$

Hence Proved.

59. Question

Prove the following trigonometric identities:

$(\sec A+\tan A-1)(\sec A-\tan A+1)=2\tan A$

Answer

$$(\sec A+\tan A-1)(\sec A-\tan A+1)$$
$$=\left[\sec A+\tan A-\left(\sec^2 A-\tan^2 A\right)\right]\left[\sec A-\tan A+\left(\sec^2 A-\tan^2 A\right)\right]$$
$$=\left[\sec A+\tan A-(\sec A-\tan A)(\sec A+\tan A)\right]\left[\sec A-\tan A+(\sec A-\tan A)(\sec A+\tan A)\right]$$
$$=(\sec A+\tan A)\left[1-(\sec A-\tan A)\right](\sec A-\tan A)\left[1+(\sec A+\tan A)\right]$$
$$=(\sec A+\tan A)(\sec A-\tan A)\left[1-\sec A+\tan A\right]\left[1+\sec A+\tan A\right]$$
$$=\left(\sec^2 A-\tan^2 A\right)\left[1-\sec A+\tan A\right]\left[1+\sec A+\tan A\right]$$
$$=1\times\left[1-\sec A+\tan A\right]\left[1+\sec A+\tan A\right]$$
$$=\left[1-\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right]\left[1+\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right]$$
$$=\left[\frac{\cos A+\sin A-1}{\cos A}\right]\left[\frac{\cos A+\sin A+1}{\cos A}\right]$$
$$=\left[\frac{(\cos A+\sin A)^2-1}{\cos^2 A}\right]$$
$$=\left[\frac{\cos^2 A+\sin^2 A+2\sin A\cos A-1}{\cos^2 A}\right]$$
$$=\left[\frac{1+2\sin A\cos A-1}{\cos^2 A}\right]$$
$$=\left[\frac{2\sin A}{\cos A}\right]$$
$$=2\tan A$$

Hence Proved.

60. Question

Prove the following trigonometric identities:

$(1+\cot A-\cos ec A)(1+\tan A+\sec A)=2$

Answer

$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$

$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)$

$= \frac{[(\sin\theta + \cos\theta) - 1][(\sin\theta + \cos\theta) + 1]}{\sin\theta \cdot \cos\theta}$

$= \frac{(\sin\theta + \cos\theta)^2 - (1)^2}{\sin\theta \cdot \cos\theta}$

$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta \cdot \cos\theta}$

$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta \cdot \cos\theta}$

$= \frac{2\sin\theta\cos\theta}{\sin\theta \cdot \cos\theta} = 2 = \text{R.H.S.}$

Hence Proved.

61. Question

Prove the following trigonometric identities:

$(\operatorname{cosec}\theta - \sec\theta)(\cot\theta - \tan\theta) = (\operatorname{cosec}\theta + \sec\theta)(\sec\theta\operatorname{cosec}\theta - 2)$

Answer

$LHS = (\operatorname{cosec}\theta - \sec\theta)(\cot\theta - \tan\theta)$

$= \left(\frac{1}{\sin\theta} - \frac{1}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\right)$

$= \left(\frac{\cos\theta - \sin\theta}{\sin\theta\cos\theta}\right)\left(\frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}\right)$

$= \left(\frac{(\cos\theta - \sin\theta)^2(\cos\theta + \sin\theta)}{\sin^2\theta\cos^2\theta}\right)$

$RHS = (\operatorname{cosec}\theta + \sec\theta)(\sec\theta\operatorname{cosec}\theta - 2)$

$= \left(\frac{1}{\sin\theta} + \frac{1}{\cos\theta}\right)\left(\frac{1}{\sin\theta} \times \frac{1}{\cos\theta} - 2\right)$

$= \left(\frac{\cos\theta + \sin\theta}{\sin\theta\cos\theta}\right)\left(\frac{1 - 2\sin\theta\cos\theta}{\sin\theta\cos\theta}\right)$

$= \left(\frac{\cos\theta + \sin\theta}{\sin\theta\cos\theta}\right)\left(\frac{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta}{\sin\theta\cos\theta}\right)$

$= \left(\frac{\cos\theta + \sin\theta}{\sin\theta\cos\theta}\right)\left(\frac{(\cos\theta - \sin\theta)^2}{\sin\theta\cos\theta}\right)$

$= \left(\frac{(\cos\theta - \sin\theta)^2(\cos\theta + \sin\theta)}{\sin^2\theta\cos^2\theta}\right)$

Therefore, LHS = RHS

Hence Proved.

62. Question

Prove the following trigonometric identities:

$(\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A) = \tan A \sec A - \cot A \operatorname{cosec} A$

Answer

To prove: $(\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A) = \tan A \sec A - \cot A \operatorname{cosec} A$

Proof: Consider LHS,(secA-cosecA)(1+tanA+cotA)We know,cosecA=1/sinA, secA=1/cosA, tanA=sinA/cosA,cotA=cosA/sinASo,

$$(\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A) = \left(\frac{1}{\cos A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= \left(\frac{\sin A - \cos A}{\cos A \sin A}\right)\left(\frac{\sin A \cos A + \sin^2 A + \cos^2 A}{\cos A \sin A}\right)$$

Using the formula $a^3 - b^3 = (a-b)(a^2+b^2+ab)$ we get,

$$= \frac{\sin^3 A - \cos^3 A}{\sin^2 A \ \cos^2 A}$$

$$RHS = \tan A \sec A - \cot A \operatorname{cosec} A$$
$$= \frac{\sin A}{\cos A} \times \frac{1}{\cos A} - \frac{\cos A}{\sin A} \times \frac{1}{\sin A}$$
$$= \frac{\sin A}{\cos^2 A} - \frac{\cos A}{\sin^2 A}$$
$$= \frac{\sin^3 A - \cos^3 A}{\sin^2 A \cos^2 A}$$
$$= \frac{\sin^3 A - \cos^3 A}{\sin^2 A \cos^2 A}$$

LHS = RHS

Hence Proved.

63. Question

Prove the following trigonometric identities:

$$\frac{\cos A \operatorname{cosec} A - \sin A \sec A}{\cos A + \sin A} = \operatorname{cosec} A - \sec A$$

Answer

$$\frac{\cos A \operatorname{cosec} A - \sin A \sec A}{\cos A + \sin A}$$
$$= \frac{\cos A \times \frac{1}{\sin A} - \sin A \frac{1}{\cos A}}{\cos A + \sin A}$$
$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\cos A}}{\cos A + \sin A}$$
$$= \frac{\frac{\cos^2 A - \sin^2 A}{\sin A \cos A}}{\cos A + \sin A}$$
$$= \frac{(\cos A - \sin A)(\cos A + \sin A)}{\sin A \cos A (\cos A + \sin A)}$$
$$= \frac{(\cos A - \sin A)}{\sin A \cos A}$$
$$= \frac{1}{\sin A} - \frac{1}{\cos A}$$
$$= \operatorname{cosec} A - \sec A$$

Hence Proved.

64. Question

Prove the following trigonometric identities:

$$\frac{\sin A}{\sec A + \tan A - 1} + \frac{\cot A}{\operatorname{cosec} A + \cot A - 1} = 1$$

Answer

$$\frac{\sin A}{\sec A+\tan A-1}+\frac{\cot A}{\operatorname{cosec} A+\cot A-1}$$
$$=\frac{\sin A}{\frac{1}{\cos A}+\frac{\sin A}{\cos A}-1}+\frac{\frac{\cos A}{\sin A}}{\frac{1}{\sin A}+\frac{\cos A}{\sin A}-1}$$
$$=\frac{\sin A\cos A}{1+\sin A-\cos A}+\frac{\sin A\cos A}{1+\cos A-\sin A}$$
$$=\sin A\cos A\left(\frac{1}{1+\sin A-\cos A}+\frac{1}{1+\cos A-\sin A}\right)$$
$$=\sin A\cos A\left(\frac{1+\cos A-\sin A+1+\sin A-\cos A}{(1+\sin A-\cos A)(1+\cos A-\sin A)}\right)$$
$$=\sin A\cos A\left(\frac{2}{1-\sin^2 A-\cos^2 A+2\sin A\cos A}\right)$$
$$=\sin A\cos A\left(\frac{2}{1-(\sin^2 A+\cos^2 A)+2\sin A\cos A}\right)$$
$$=\sin A\cos A\left(\frac{2}{1-1+2\sin A\cos A}\right)$$
$$=\sin A\cos A\left(\frac{1}{\sin A\cos A}\right)$$
$$=1$$

Hence Proved.

65. Question

Prove the following trigonometric identities:

$$\frac{\tan A}{(1+\tan^2 A)^2}+\frac{\cot A}{(1+\cot^2 A)^2}=\sin A\cos A$$

Answer

$$\frac{\tan A}{(1+\tan^2 A)^2}+\frac{\cot A}{(1+\cot^2 A)^2}=\frac{\tan A}{(\sec^2 A)^2}+\frac{\cot A}{(\operatorname{cosec}^2 A)^2}$$
$$=\frac{\sin A}{\cos A}\times\cos^4 A+\frac{\cos A}{\sin A}\times\sin^4 A$$
$$=\sin A\cos^3 A+\cos A\sin^3 A$$
$$=\sin A\cos A\left(\cos^2 A+\sin^2 A\right)$$
$$=\sin A\cos A\times 1$$
$$=\sin A\cos A$$

Hence Proved.

66. Question

Prove the following trigonometric identities:

$$\sec^4 A(1-\sin^4 A)-2\tan^2 A=1$$

Answer

$$\sec^4 A(1-\sin^4 A)-2\tan^2 A$$
$$=\sec^4 A-\sec^4 A\sin^4 A-2\tan^2 A$$
$$=\sec^4 A-\frac{\sin^4 A}{\cos^4 A}-2\tan^2 A$$
$$=\left(\sec^2 A\right)^2-\tan^4 A-2\tan^2 A$$
$$=\left(1+\tan^2 A\right)^2-\tan^4 A-2\tan^2 A$$
$$=1+\tan^4 A+2\tan^2 A-\tan^4 A-2\tan^2 A$$
$$=1$$

Hence Proved.

67. Question

Prove the following trigonometric identities:

$$\frac{\cot^2 A(\sec A-1)}{1+\sin A}=\sec^2 A\left(\frac{1-\sin A}{1+\sec A}\right)$$

Answer

$$\frac{\cot^2 A(\sec A-1)}{1+\sin A}=\frac{\frac{\cos^2 A}{\sin^2 A}\left(\frac{1}{\cos A}-1\right)}{1+\sin A}$$

$$=\frac{\frac{\cos^2 A}{\sin^2 A}\left(\frac{1-\cos A}{\cos A}\right)}{1+\sin A}$$

$$=\frac{\frac{\cos A}{1-\cos^2 A}(1-\cos A)}{1+\sin A}$$

$$=\frac{\frac{\cos A}{(1-\cos A)(1+\cos A)}(1-\cos A)}{1+\sin A}$$

$$=\frac{\cos A}{(1+\cos A)(1+\sin A)} \quad .....(1)$$

$$RHS=\frac{1}{\cos^2 A}\left(\frac{1-\sin A}{1+\frac{1}{\cos A}}\right)$$

$$=\frac{1}{\cos^2 A}\frac{(1-\sin A)\cos A}{(1+\cos A)}$$

$$=\frac{1}{\cos A}\frac{(1-\sin A)}{(1+\cos A)}\times\frac{(1+\sin A)}{(1+\sin A)}$$

$$=\frac{1}{\cos A}\frac{(1-\sin^2 A)}{(1+\cos A)(1+\sin A)}$$

$$=\frac{1}{\cos A}\frac{\cos^2 A}{(1+\cos A)(1+\sin A)}$$

$$=\frac{\cos A}{(1+\cos A)(1+\sin A)} \quad ....(2)$$

From (1) and (2) we get

LHS = RHS

Hence Proved.

68. Question

Prove the following trigonometric identities:

$$(1+\cot A+\tan A)(\sin A-\cos A)=\frac{\sec A}{\operatorname{cosec}^2 A}-\frac{\operatorname{cosec} A}{\sec^2 A}=\sin A\tan A-\cot A\cos A$$

Answer

To Prove: $(1+\cot A+\tan A)(\sin A-\cos A)=\frac{\sec A}{\operatorname{cosec}^2 A}-\frac{\operatorname{cosec} A}{\sec^2 A}=\sin A\tan A-\cot A\cos A$

Proof: Consider the LHS,

$$\Rightarrow \frac{\sec A}{\operatorname{cosec}^2 A}-\frac{\operatorname{cosec} A}{\sec^2 A}=\left(\frac{1}{\cos A}\times\sin^2 A\right)-\left(\frac{1}{\sin A}\times\cos^2 A\right)$$

= sinA - cosA+ cotA sinA - cotA cosA + tanA sinA - tanA cosA

Use the formula:

$$\tan\theta=\frac{\sin\theta}{\cos\theta}\ and\ \cot\theta=\frac{\cos\theta}{\sin\theta}$$

$$=sinA-cosA+\left(\frac{cosA}{sinA}\times sinA\right)-\left(\frac{cosA}{sinA}\times cosA\right)+\left(\frac{sinA}{cosA}\times sinA\right)-\left(\frac{sinA}{cosA}\times cosA\right)$$

$$=sinA-cosA+\cos A-\frac{\cos^2 A}{\sin A}+\frac{\sin^2 A}{\cos A}-\sin A$$

$$= \frac{sin^2 A}{cos A} - \frac{cos^2 A}{sin A}$$

We know:

$$\sin\theta = \frac{1}{\text{cosec}\theta} \; and \; \cos\theta = \frac{1}{\sec\theta}$$

So,

$$(1 + \cot A + \tan A)(\sin A - \cos A) = \frac{\sec A}{\text{cosec}^2 A} - \frac{\text{cosec} A}{sec^2 A}$$

Again use the formula:

$$\sin\theta = \frac{1}{\text{cosec}\theta} \; and \; \cos\theta = \frac{1}{\sec\theta}$$

So,

$$\Rightarrow \frac{sec A}{cosec^2 A} - \frac{cosec A}{sec^2 A} = \left(\frac{1}{cos A} \times sin^2 A\right) - \left(\frac{1}{sin A} \times cos^2 A\right)$$

$$\Rightarrow \frac{sec A}{cosec^2 A} - \frac{cosec A}{sec^2 A} = \frac{sin A \times \sin A}{cos A} - \frac{cos A \cos A}{sin A}$$

Use the formula:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \; and \; \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\Rightarrow \frac{sec A}{cosec^2 A} - \frac{cosec A}{sec^2 A} = \sin A \tan A - \cot A \cos A$$

Therefore, $(1 + cot A + tan A)(sin A - cos A) = \frac{sec A}{cosec^2 A} - \frac{cosec A}{sec^2 A} = sin A \, tan A - cot A \, cos A$

Hence Proved.

69. Question

Prove the following trigonometric identities:

$\sin^2 A \cos^2 B - \cos^2 A \sin^2 B = \sin^2 A - \sin^2 B$

Answer

To prove: $\sin^2 A \cos^2 B - \cos^2 A \sin^2 B = \sin^2 A - \sin^2 B$ **Proof:** Take LHS,Use the identity $\sin^2\theta+\cos^2\theta=1$

$\sin^2 A \cos^2 B - \cos^2 A \sin^2 B = \sin^2 A(1 - \sin^2 B) - (1 - \sin^2 A)\sin^2 B$ $= \sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B$

$= \sin^2 A - \sin^2 B$ = RHSHence Proved

70. Question

Prove the following trigonometric identities:

$\frac{\cot A + \tan B}{\cot B + \tan A} = \cot A \tan B$

Answer

Use the formula $\tan\theta = \frac{\sin\theta}{\cos\theta}\ and\ \cot\theta = \frac{\cos\theta}{\sin\theta}$

$$\text{L.H.S} = \frac{\cot A + \tan B}{\cot B + \tan A}$$
$$= \frac{\frac{\cos A}{\sin A} + \frac{\sin B}{\cos B}}{\frac{\cos B}{\sin B} + \frac{\cos A}{\cot A}}$$
$$= \frac{\frac{\cos A \cot B \sin A \sin B}{\sin A \cos B}}{\frac{\cos A \cos B + \sin A \sin B}{\cos A \sin B}}$$
$$= \frac{\cos A \cos B + \sin A \sin B}{\sin A \cos B} \times \frac{\cos A \sin B}{\cos A \cos B + \sin A \sin B}$$
$$= \frac{\cos A \sin B}{\sin A \cos B}$$
$$= \cot A \tan B$$
$$= \text{R.H.S}$$

Hence Proved.

71. Question

Prove the following trigonometric identities:

$$\frac{\tan A + \tan B}{\cot A + \cot B} = \tan A \tan B$$

Answer

$$\frac{\tan A + \tan B}{\cot A + \cot B} = \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B}}$$
$$= \frac{\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}}{\frac{\sin B \cos A + \cos B \sin A}{\sin A \sin B}}$$
$$= \frac{\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}}{\frac{\sin B \cos A + \cos B \sin A}{\sin A \sin B}}$$
$$= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B} \times \frac{\sin A \sin B}{\sin B \cos A + \cos B \sin A}$$
$$= \frac{\sin A \sin B}{\cos A \cos B}$$
$$= \tan A \tan B$$

Hence Proved.

72. Question

Prove the following trigonometric identities:

$$\cot^2 A \operatorname{cosec}^2 B - \cot^2 B \operatorname{cosec}^2 A = \cot^2 A - \cot^2 B$$

Answer

$$\cot^2 A \operatorname{cosec}^2 B - \cot^2 B \operatorname{cosec}^2 A$$
$$= \cot^2 A\left(1 + \cot^2 B\right) - \cot^2 B\left(1 + \cot^2 A\right)$$
$$= \cot^2 A + \cot^2 A \cot^2 B - \cot^2 B - \cot^2 A \cot^2 B$$
$$= \cot^2 A - \cot^2 B$$

Hence Proved.

73. Question

Prove the following trigonometric identities:

$$\tan^2 A \sec^2 B - \sec^2 A \tan^2 B = \tan^2 A - \tan^2 B$$

Answer

$$\tan^2 A\sec^2 B-\sec^2 A\tan^2 B$$
$$=\tan^2 A\left(1+\tan^2 B\right)-\left(1+\tan^2 A\right)\tan^2 B$$
$$=\tan^2 A+\tan^2 A\tan^2 B-\tan^2 B-\tan^2 A\tan^2 B$$
$$=\tan^2 A-\tan^2 B$$

Hence Proved.

74. Question

If $x=a\sec\theta+b\tan\theta$ and $y=a\tan\theta+b\sec\theta$, prove that $x^2-y^2=a^2-b^2$

$x^2-y^2=a^2-b^2$

Answer

$$x=a\sec\theta+b\tan\theta$$
$$\Rightarrow x^2=\left(a\sec\theta+b\tan\theta\right)^2=a^2\sec^2\theta+b^2\tan^2\theta+2ab\sec\theta\tan\theta$$
$$y=a\tan\theta+b\sec\theta$$
$$\Rightarrow y^2=\left(a\tan\theta+b\sec\theta\right)^2=a^2\tan^2\theta+b^2\sec^2\theta+2ab\tan\theta\sec\theta$$

Now,

$$x^2-y^2=a^2\sec^2\theta+b^2\tan^2\theta+2ab\sec\theta\tan\theta-a^2\tan^2\theta-b^2\sec^2\theta-2ab\tan\theta\sec\theta$$
$$x^2-y^2=a^2\sec^2\theta+b^2\tan^2\theta-a^2\tan^2\theta-b^2\sec^2\theta$$
$$x^2-y^2=a^2\left(\sec^2\theta-\tan^2\theta\right)-b^2\left(\sec^2\theta-\tan^2\theta\right)$$
$$x^2-y^2=\left(a^2-b^2\right)\left(\sec^2\theta-\tan^2\theta\right)$$
$$x^2-y^2=\left(a^2-b^2\right)\times 1$$
$$x^2-y^2=a^2-b^2$$

Hence Proved.

75. Question

$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$ and $\frac{x}{a}\sin\theta-\frac{y}{b}\cos\theta=1$, prove that $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$.

Answer

$$\left(\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta\right)^2+\left(\frac{x}{a}\sin\theta-\frac{y}{b}\cos\theta\right)^2=2$$
$$\frac{x^2}{a^2}\cos^2\theta+\frac{y^2}{b^2}\sin^2\theta+2\frac{xy}{ab}\cos\theta\sin\theta+\frac{x^2}{a^2}\sin^2\theta+\frac{y^2}{b^2}\cos^2\theta-2\frac{xy}{ab}\sin\theta\cos\theta=2$$
$$\frac{x^2}{a^2}\cos^2\theta+\frac{x^2}{a^2}\sin^2\theta+\frac{y^2}{b^2}\sin^2\theta+\frac{y^2}{b^2}\cos^2\theta=2$$
$$\frac{x^2}{a^2}\left(\cos^2\theta+\sin^2\theta\right)+\frac{y^2}{b^2}\left(\sin^2\theta+\cos^2\theta\right)=2$$
$$\frac{x^2}{a^2}\times 1+\frac{y^2}{b^2}\times 1=2$$
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$$

Hence Proved.

76. Question

If cosecθ - sinθ = a^3, secθ - cosθ = b^3, prove that $a^2b^2(a^2 + b^2)$.

Answer

$$\frac{1}{\sin\theta}-\sin\theta=a^3$$
$$\frac{1-\sin^2\theta}{\sin\theta}=a^3$$
$$\frac{\cos^2\theta}{\sin\theta}=a^3$$
$$a^3=\frac{\cos^2\theta}{\sin\theta}$$
$$a=\frac{\cos^{2/3}\theta}{\sin^{1/3}\theta}$$
$$\Rightarrow\quad a^2=\frac{\cos^{4/3}\theta}{\sin^{2/3}\theta}\qquad \text{....(1)}$$

Similarly we can see that,

$$\sec\theta - \cos\theta = b^3$$

$$\frac{1}{\cos\theta} - \cos\theta = b^3$$

$$\frac{1 - \cos^2\theta}{\cos\theta} = b^3$$

$$b^3 = \frac{\sin^2\theta}{\cos\theta}$$

$$b = \frac{\sin^{2/3}\theta}{\cos^{1/3}\theta}$$

$$b^2 = \frac{\sin^{4/3}\theta}{\cos^{2/3}\theta} \ldots\ldots(2)$$

From (1) and (2), we get

$$a^2b^2\left(a^2+b^2\right) = \cos^{\frac{4}{3}-\frac{2}{3}}\theta \sin^{\frac{4}{3}-\frac{2}{3}}\theta\left(\frac{\cos^2\theta+\sin^2\theta}{\sin^{2/3}\theta\cos^{2/3}\theta}\right)$$

$$= \cos^{\frac{2}{3}}\theta \sin^{\frac{2}{3}}\theta\left(\frac{1}{\sin^{2/3}\theta\cos^{2/3}\theta}\right)$$

$$= 1$$

Hence Proved.

77. Question

If $a\cos^3\theta + 3a\cos\theta\sin^2\theta = m$, $a\sin^3\theta + 3a\cos^2\theta\sin\theta = n$, prove that $(m+n)^{2/3} + (m-n)^{2/3} = 2a^{2/3}$.

Answer

$$(m+n)^{2/3} + (m-n)^{2/3} = 2a^{2/3}$$

$$= \left(a\cos^3\theta + 3a\cos\theta\sin^2\theta + a\sin^3\theta + 3a\cos^2\theta\sin\theta\right)^{2/3}$$

$$+\left(a\cos^3\theta + 3a\cos\theta\sin^2\theta + a\sin^3\theta + 3a\cos^2\theta\sin\theta\right)^{2/3}$$

$$= a^{2/3}\left(\cos^3\theta + 3\cos\theta\sin^2\theta + \sin^3\theta + 3\cos^2\theta\sin\theta\right)^{2/3}$$

$$+ a^{2/3}\left(a\cos^3\theta + 3a\cos\theta\sin^2\theta + a\sin^3\theta + 3a\cos^2\theta\sin\theta\right)^{2/3}$$

$$= a^{2/3}\left[(\cos\theta+\sin\theta)^3\right]^{2/3} + a^{2/3}\left[(\cos\theta-\sin\theta)^3\right]^{2/3}$$

$$= a^{2/3}(\cos\theta+\sin\theta)^2 + a^{2/3}(\cos\theta-\sin\theta)^2$$

$$= a^{2/3}\left(\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta\right) + a^{2/3}\left(\cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta\right)$$

$$= a^{2/3}\left[(1+2\sin\theta\cos\theta)+(1-2\sin\theta\cos\theta)\right]$$

$$= a^{2/3}\times 2$$

$$= a^{2/3}$$

Hence Proved.

78. Question

If $x = a\cos^3\theta$, $y = b\sin^3\theta$, prove that $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$.

Answer

$$x = a\cos^3\theta \quad\Rightarrow\quad \frac{x}{a} = \cos^3\theta$$

$$y = b\sin^3\theta \quad\Rightarrow\quad \frac{y}{b} = \sin^3\theta$$

$$\text{Now, } \left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = \left(\cos^3\theta\right)^{2/3} + \left(\sin^3\theta\right)^{2/3}$$

$$= \cos^2\theta + \sin^2\theta$$

$$= 1$$

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$$

79. Question

If $3\sin\theta + 5\cos\theta = 5$, prove that $5\sin\theta - 3\cos\theta = \pm 3$.

Answer

$$\begin{aligned} & 3\sin\theta + 5\cos\theta = 5 \\ \Rightarrow\ & 3\sin\theta = 5 - 5\cos\theta \\ \Rightarrow\ & 3\sin\theta = 5(1-\cos\theta) \\ \Rightarrow\ & 3\sin\theta = \frac{5(1-\cos\theta)\times(1+\cos\theta)}{(1+\cos\theta)} \\ \Rightarrow\ & 3\sin\theta = \frac{5(1-\cos^2\theta)}{(1+\cos\theta)} \\ \Rightarrow\ & 3\sin\theta = \frac{5\sin^2\theta}{(1+\cos\theta)} \\ \Rightarrow\ & 3 = \frac{5\sin\theta}{(1+\cos\theta)} \\ \Rightarrow\ & 3 + 3\cos\theta = 5\sin\theta \\ \Rightarrow\ & 3 = 5\sin\theta - 3\cos\theta \end{aligned}$$

Hence Proved.

80. Question

If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, prove that $a^2 + b^2 = m^2 + n^2$

Answer

$$\begin{aligned} m^2 + n^2 &= (a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2 \\ &= a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\cos\theta\sin\theta \\ &= a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) \\ &= a^2\times 1 + b^2\times 1 \\ &= a^2 + b^2 \end{aligned}$$

Hence Proved.

81. Question

If $\operatorname{cosec}\theta + \cot\theta = m$ and $\operatorname{cosec}\theta - \cot\theta = n$, prove that $mn = 1$.

Answer

$$\begin{aligned} & \operatorname{cosec}\theta + \cot\theta = m \\ & \operatorname{cosec}\theta - \cot\theta = n \\ & mn = (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) \\ & mn = (\operatorname{cosec}^2\theta - \cot^2\theta) \\ & mn = 1 \end{aligned}$$

Hence Proved.

82. Question

If $\cos A + \cos^2 A = 1$, prove that $\sin^2 A + \sin^4 A = 1$

Answer

Consider, $\cos A + \cos^2 A = 1 \Rightarrow \cos A = 1 - \cos^2 A$ As we know $1 - \cos^2 A = \sin^2 A \Rightarrow \cos A = \sin^2 A$ .... (1)Now $\sin^2 A + \sin^4 A = \sin^2 A + (\sin^2 A)^2$ From 1$\sin^2 A + \sin^4 A = \sin^2 A + (\cos A)^2$ $= \sin^2 A + \cos^2 A$ $= 1$

Hence Proved.

83. Question

Prove that:

(i) $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = 2\operatorname{cosec}\theta$

(ii) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$

(iii) $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = 2\operatorname{cosec}\theta$

(iv) $\frac{\sec\theta - 1}{\sec\theta + 1} = \left(\frac{\sin\theta}{1+\cos\theta}\right)^2$

Answer

(i)

$$\sqrt{\frac{\sec\theta-1}{\sec\theta+1}}+\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}=\sqrt{\frac{\frac{1}{\cos\theta}-1}{\frac{1}{\cos\theta}+1}}+\sqrt{\frac{\frac{1}{\cos\theta}+1}{\frac{1}{\cos\theta}-1}}$$

$$=\sqrt{\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cos\theta}}}+\sqrt{\frac{\frac{1+\cos\theta}{\cos\theta}}{\frac{1-\cos\theta}{\cos\theta}}}$$

$$=\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}+\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$$

$$=\sqrt{\frac{1-\cos\theta}{1+\cos\theta}\times\frac{1-\cos\theta}{1-\cos\theta}}+\sqrt{\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1-\cos\theta}{1+\cos\theta}}$$

$$=\sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}+\sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}}$$

$$=\sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}+\sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}}$$

$$=\sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}+\sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}$$

$$=\frac{1-\cos\theta}{\sin\theta}+\frac{1+\cos\theta}{\sin\theta}$$

$$=\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}+\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}$$

$$=\frac{1}{\sin\theta}+\frac{1}{\sin\theta}$$

$$=\frac{2}{\sin\theta}$$

$$=2\operatorname{cosec}\theta$$

Hence Proved.

(ii)

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=\sqrt{\frac{1+\sin\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}}$$

$$=\sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}+\sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$

$$=\sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}+\sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$

$$=\frac{1+\sin\theta}{\cos\theta}+\frac{1-\sin\theta}{\cos\theta}$$

$$=\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}$$

$$=\frac{2}{\cos\theta}$$

$$=2\sec\theta$$

Hence Proved.

(iii)

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}+\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\sqrt{\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}}+\sqrt{\frac{1-\cos\theta}{1+\cos\theta}\times\frac{1-\cos\theta}{1-\cos\theta}}$$

$$=\sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}}+\sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$$

$$=\sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}+\sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$$

$$=\frac{1+\cos\theta}{\sin\theta}+\frac{1-\cos\theta}{\sin\theta}$$

$$=\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}+\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}$$

$$=\frac{2}{\sin\theta}$$

$$=2\operatorname{cosec}\theta$$

Hence Proved.

(iv)

$$\frac{\sec\theta - 1}{\sec\theta + 1} = \frac{\frac{1}{\cos\theta} - 1}{\frac{1}{\cos\theta} + 1}$$

$$= \frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cos\theta}}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$

$$= \frac{1-\cos\theta}{1+\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$$

$$= \frac{1-\cos^2\theta}{(1+\cos\theta)^2}$$

$$= \frac{\sin^2\theta}{(1+\cos\theta)^2}$$

$$= \left(\frac{\sin\theta}{1+\cos\theta}\right)^2$$

Hence Proved.

84. Question

If $\cos\theta + \cos^2\theta = 1$, prove that

Answer

$$\sin^{12}\theta + 3\sin^{10}\theta + 3\sin^8\theta + \sin^6\theta + 2\sin^4\theta + 2\sin^2\theta - 2 = 1$$

$$\cos\theta + \cos^2\theta = 1$$

$$\cos = 1 - \cos^2\theta$$

$$\cos = \sin^2\theta - (i)$$

Now, $\text{Sin}^{12}\theta + 3\sin^{10}\theta + 3\sin^8\theta + \sin^6\theta + 2\sin^4\theta + 2\sin^2\theta - 2$

$$= (\sin^4\theta)^3 + \sin^4\theta - \sin^2\theta[\sin^4\theta + \sin^2\theta]$$

$$+ (\sin^2\theta)^3 + 2(\sin^2\theta)^2 + 2\sin^2\theta - 2$$

Using $(a+b)^3 = a^3 + b^3 + 3(a+b)$and

Also from (i) $\sin^2\theta\cos^2$

$(\sin^4\theta + \sin^2\theta)^3 + 2(\cos\theta)^2 + 2\cos\theta - 2.$

$((\sin^2\theta)^2 + \sin^2\theta) + 2\cos^2\theta + 2\cos\theta - 2$

$(\cos^2 + \sin^2\theta)^3 + 2\cos^2\theta + 2\cos\theta - 2$

$(\cos)^3 + 2\cos^2\theta + 2\sin^2\theta - 2$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$1 + 2(\sin^2\theta + \cos^2\theta) - 2$

$1 + 2(1) - 2 = 1$

Hence Proved.

85. Question

Given that:

$$(1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma) = (1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)$$

Show that one of the values of each member of this equality is $\sin\alpha\sin\beta\sin\gamma$

Answer

L.H.S

we know that $1+\cos\theta = 1+\cos^2 \theta/2 - \sin^2 \theta/2$

$= 2\cos^2 \theta/2.$

$\therefore \Rightarrow 2\cos^2 \alpha/2 - 2\cos^2 \beta/2 . 2\cos^2 \chi/2 \ldots(i)$

Multiply (i) with $\sin\alpha\sin\beta\sin\chi$ and divide with same

we get $\frac{8\cos^2 \alpha/2 \cos^2 \beta/2 \cos^2 \chi/2}{\sin\alpha\sin\beta\sin\chi} \times \sin\alpha\sin\beta\sin\chi$

$\Rightarrow \frac{8\cos^2 \alpha/2 \cos^2 \beta/2 \cos^2 \chi/2 \times \sin\alpha\sin\beta\sin\chi}{2.2\theta\cos \alpha/2 \cos \beta/2 \cos \chi/2 \sin \beta/2 \sin \beta/2 \cos \chi/2}$

$\Rightarrow \sin\alpha\sin\beta\sin\chi \cot \alpha/2 \cot \beta/2 \cot \chi/2$

R.H.S $(1-\cos\alpha)(1-\cos\beta)(1\cos\chi)$

we know that $1-\cos\theta = 1-\cos^2 \theta/2 + \sin^2 \theta/2 = 2\sin^2 \theta/2$

$\Rightarrow 2\sin^2 \alpha/2, 2\sin^2 \beta/2 2\sin^2 \chi/2$

Multiply and divide by $\sin\alpha\sin\beta\sin\chi$ we get

$\frac{2\sin^2 \alpha/2, 2\sin^2 \beta/2 2\sin^2 \chi/2 . \sin\alpha\sin\beta\sin\chi}{\sin\alpha\sin\beta\sin\chi}$

$\Rightarrow \frac{2\sin^2 \alpha/2, 2\sin^2 \beta/2 2\sin^2 \chi/2 . \sin\alpha\sin\beta\sin\chi}{2\sin \alpha/2 \cos \alpha/2 2\sin \beta/2 \cos \beta/2 2\sin \alpha/2 \cos \alpha/2}$

$\Rightarrow \text{Tan}\, \alpha/2\, \text{Tan}\, \beta/2\, \text{Tan}\, \chi/2 \sin\alpha\sin\beta\sin\chi$

hence $\sin\alpha\sin\beta\sin\chi$ is the member of equality

Hence Proved.

86. Question

If $\sin\theta + \cos\theta = x$, prove that $\sin^6\theta + \cos^6\theta = \frac{4-3(x^2-1)^2}{4}$

Answer

$\text{Sin}\theta + \text{Cos}\theta = x$

Squaringon both sides

$(\text{Sin}\theta + \text{Cos})^2 = x^2$

$\Rightarrow \text{Sin}^2\theta + \text{Cos}^2\theta + 2\text{Sin}\theta\text{Cos}\theta = x^2$

$\therefore \text{Sin}\theta\text{Cos}\theta = \frac{x^2-1}{2} \quad \ldots(1)$

We know $\text{Sin}^2\theta + \text{Cos}^2\theta = 1$

cobingon both since

$(\sin^2\theta + \cos^2\theta)^3 = (1)^3$

$\text{Sin}^6\theta + \text{Cos}^6\theta + 3\text{Sin}\theta\,\text{Cos}^2(\text{Sin}^2\theta + \text{Cos}^2\theta) = 1$

$\Rightarrow \text{Sin}^6\theta + \text{Cos}^2 = 1 - 3\text{Sin}^2\theta\text{Cos}^2\theta$

$= 1 - \frac{3(x^2-1)^2}{4}$ form - (1)

$\therefore \text{Sin}^6\theta + \text{Cos}^6\theta = \frac{4-3(x^2-1)^2}{4}$

Hence proved

Hence Proved.

87. Question

If $x = a\sec\theta\cos\phi$, $y = b\sec\theta\cos\phi$, and $z = c\tan\theta$, show that $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

Answer

$$x = a\sec\theta\cos\phi \quad \Rightarrow x^2 = a^2\sec^2\theta\cos^2\phi$$

$$\therefore \quad \frac{x^2}{a^2} = \sec^2\theta\cos^2\phi \qquad ....(1)$$

$$y = b\sec\theta\sin\phi \quad \Rightarrow y^2 = b^2\sec^2\theta\sin^2\phi$$

$$\therefore \quad \frac{y^2}{a^2} = \sec^2\theta\sin^2\phi \qquad ....(2)$$

$$z = c\tan\theta \quad \Rightarrow \quad z^2 = c^2\tan^2\theta$$

$$\therefore \quad \frac{z^2}{c^2} = \tan^2\theta \qquad ....(3)$$

$$Now, \quad \frac{x^2}{a^2} + \frac{y^2}{a^2} - \frac{z^2}{c^2} = \sec^2\theta\cos^2\phi + \sec^2\theta\sin^2\phi - \tan^2\theta$$

$$= \sec^2\theta\left(\cos^2\phi + \sin^2\phi\right) - \tan^2\theta$$

$$= \sec^2\theta \times 1 - \tan^2\theta$$

$$= \sec^2\theta - \tan^2\theta = 1$$

$$hence, \quad \frac{x^2}{a^2} + \frac{y^2}{a^2} - \frac{z^2}{c^2} = 1$$

Hence Proved.

Exercise 6.2

1. Question

If $\cos\theta = \frac{4}{5}$, find all other trigonometric ratios of angle θ.

Answer

$\sin\theta = 3/5, \tan\theta = 3/4$, $\sec\theta = 5/4,$

$$\cos\theta = \frac{4}{5} \qquad \Rightarrow \sin\theta = \frac{3}{5}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{4/5} = \frac{5}{4}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3/5}{4/5} = \frac{3}{4}$$

Hence Proved.

2. Question

If $\sin\theta = \frac{1}{\sqrt{2}}$, find all other trigonometric ratios of angle θ

Answer

$$\cos\theta = \frac{1}{\sqrt{2}} \quad \Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1/\sqrt{2}}{1/\sqrt{2}} = 1$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{1/\sqrt{2}} = \sqrt{2}$$

$$co\sec\theta = \frac{1}{\sin\theta} = \frac{1}{1/\sqrt{2}} = \sqrt{2}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{1} = 1$$

3. Question

If $\tan\theta = \frac{1}{\sqrt{2}}$, find the value of $\frac{\cos ec^2\theta - \sec^2\theta}{\cos ec^2\theta + \cot^2\theta}$

Answer

$$Given,\ \tan\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{P}{B} = \frac{1}{\sqrt{2}}$$

$$Let\ P = k\ and\ B = \sqrt{2}k$$

$$Then,$$

$$H = \sqrt{P^2 + B^2}$$

$$= \sqrt{k^2 + 2k^2}$$

$$= \sqrt{3k^2}$$

$$= \sqrt{3}k$$

$$Hence,$$

$$\sin\theta = \frac{P}{H}$$

$$\sin\theta = \frac{k}{\sqrt{3}k} = \frac{1}{\sqrt{3}}$$

$$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \cot^2\theta} = \frac{\operatorname{cosec}^2\theta\left(1 - \frac{\sec^2\theta}{\operatorname{cosec}^2\theta}\right)}{\operatorname{cosec}^2\theta\left(1 + \frac{\cot^2\theta}{\operatorname{cosec}^2\theta}\right)}$$

$$= \frac{\left(1 - \frac{\sin^2\theta}{\cos^2\theta}\right)}{\left(1 + \sin^2\theta \times \cot^2\theta\right)}$$

$$= \frac{\left(1 - \tan^2\theta\right)}{\left(1 + \sin^2\theta \times \frac{1}{\tan^2\theta}\right)}$$

$$= \frac{1 - \left(\frac{1}{\sqrt{2}}\right)^2}{\left(1 + \left(\frac{1}{\sqrt{3}}\right)^2 \times \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2}\right)} = \frac{1 - \frac{1}{2}}{\left(1 + \left(\frac{1}{\sqrt{3}}\right)^2 \times \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2}\right)}$$

$$= \frac{1/2}{1 + \frac{2}{3}} = \frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$$

4. Question

If $\tan\theta = \frac{3}{4}$, find the value of $\frac{1 - \cos\theta}{1 + \cos\theta}$

Answer

$$\tan\theta = \frac{3}{4} \qquad \Rightarrow \cos\theta = \frac{4}{5}$$

$$Now,\ \frac{1 - \cos\theta}{1 + \cos\theta} = \frac{1 - \frac{4}{5}}{1 + \frac{4}{5}} = \frac{1/5}{9/5} = \frac{1}{9}\ \frac{1 - \cos\theta}{1 + \cos\theta}$$

5. Question

If $\tan\theta = \frac{12}{5}$, find the value of $\frac{1+\sin\theta}{1-\sin\theta}$

Answer

$$\frac{1+\sin\theta}{1-\sin\theta} = \frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}$$
$$= \frac{(1+\sin\theta)^2}{1-\sin^2\theta}$$
$$= \frac{(1+\sin\theta)^2}{\cos^2\theta}$$
$$= \sec^2\theta + \tan^2\theta$$
$$= \tan^2\theta + 1 + \tan^2\theta$$
$$= 2\tan^2\theta + 1$$
$$= 2\times\left(\frac{12}{5}\right)^2 + 1 \qquad \left(\because \tan\theta = \frac{12}{5}\right)$$
$$= \frac{288}{25} + 1$$
$$= \frac{288+25}{25} = \frac{313}{25}$$

6. Question

If $\cot\theta = \frac{1}{\sqrt{3}}$, find the value of $\frac{1-\cos^2\theta}{2-\sin^2\theta}$

Answer

Given, $\cot\theta = \frac{1}{\sqrt{3}}$

Now,

$$\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{\sin^2\theta}{1+\cos^2\theta}$$
$$= \frac{1}{\frac{1+\cos^2\theta}{\sin^2\theta}}$$
$$= \frac{1}{\frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta}}$$
$$= \frac{1}{\mathrm{cosec}^2\theta + \cot^2\theta}$$
$$= \frac{1}{1+\cot^2\theta + \cot^2\theta}$$
$$= \frac{1}{1+2\cot^2\theta}$$
$$= \frac{1}{1+2\times\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{1}{1+2\times\frac{1}{3}} = \frac{1}{1+\frac{2}{3}} = \frac{1}{\frac{5}{3}} = \frac{3}{5}$$

7. Question

If $\mathrm{cosec}A = \sqrt{2}$, find the value of $\frac{2\sin^2 A + 3\cot^2 A}{4(\tan^2 A - \cos^2 A)}$

Answer

Given, $\mathrm{cosec}A = \sqrt{2} \quad \Rightarrow \cot^2 A = \mathrm{cosec}^2 A - 1 = \left(\sqrt{2}\right)^2 - 1 = 2 - 1 = 1$

and, $\tan^2 A = \frac{1}{\cot^2 A} = \frac{1}{1} = 1$

$\Rightarrow \quad \sin^2 A = \frac{1}{\mathrm{cosec}^2 A} = \frac{1}{\left(\sqrt{2}\right)^2} = \frac{1}{2}$

$\Rightarrow \quad \cos^2 A = 1 - \sin^2 A = 1 - \left(\frac{1}{2}\right)^2 = 1 - \frac{1}{4} = \frac{3}{4}$

Now,

$$\frac{2\sin^2 A + 3\cot^2 A}{4(\tan^2 A - \cos^2 A)} = \frac{2\times\frac{1}{2} + 3\times 1}{4\left(1-\frac{3}{4}\right)} = \frac{1+3}{4\times\frac{1}{4}} = 4$$

8. Question

If $\cot\theta = \sqrt{3}$, find the value of $\frac{\operatorname{cosec}^2\theta + \cot^2\theta}{\operatorname{cosec}^2\theta - \sec^2\theta}$

Answer

$$\textit{Given,}\quad \cot\theta = \sqrt{3}$$

$$\operatorname{cosec}^2\theta = 1 + \cot^2\theta = 1 + \left(\sqrt{3}\right)^2 = 1 + 3 = 4$$

$$\textit{and}\quad \cot\theta = \sqrt{3} \quad \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \sec^2\theta = 1 + \tan^2\theta = 1 + \left(\frac{1}{\sqrt{3}}\right)^2 = 1 + \frac{1}{3} = \frac{4}{3}$$

Now,

$$\frac{\operatorname{cosec}^2\theta + \cot^2\theta}{\operatorname{cosec}^2\theta - \sec^2\theta} = \frac{4+3}{4-\frac{4}{3}} = \frac{7}{8/3} = \frac{21}{8}$$

9. Question

If $3\cos\theta = 1$, find the value of $\frac{6\sin^2\theta + \tan^2\theta}{4\cos\theta}$

Answer

$$3\cos\theta = 1 \quad \Rightarrow \cos\theta = \frac{1}{3}$$

$$\Rightarrow \sin^2\theta = 1 - \cos^2\theta = 1 - \left(\frac{1}{3}\right)^2 = 1 - \frac{1}{9} = \frac{8}{9}$$

$$\Rightarrow \sin\theta = \frac{2\sqrt{2}}{3}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2\sqrt{2}/3}{1/3} = 2\sqrt{2}$$

$$\frac{6\sin^2\theta + \tan^2\theta}{4\cos\theta} = \frac{6\times\frac{8}{9}+8}{4\times\frac{1}{3}} = \frac{\frac{16}{3}+8}{\frac{4}{3}} = \frac{\frac{30}{3}}{\frac{4}{3}} = \frac{30}{4} = 7\frac{1}{2}$$

10. Question

If $\sqrt{3}\tan\theta = 3\sin\theta$, find the value of $\sin^2\theta - \cos^2\theta$

Answer

$$\sqrt{3}\tan\theta = 3\sin\theta$$

$$\sqrt{3}\frac{\sin\theta}{\cos\theta} = \sqrt{3}\times\sqrt{3}\sin\theta$$

$$\frac{1}{\cos\theta} = \sqrt{3}$$

$$\cos\theta = \frac{1}{\sqrt{3}}$$

$$\begin{aligned} \textit{Now,}\quad \sin^2\theta - \cos^2\theta &= 1 - \cos^2\theta - \cos^2\theta \\ &= 1 - 2\cos^2\theta \\ &= 1 - 2\times\left(\frac{1}{\sqrt{3}}\right)^2 \\ &= 1 - 2\times\frac{1}{3} = \frac{1}{3} \end{aligned}$$

11. Question

If $\operatorname{cosec}\theta = \frac{13}{12}$, find the value of $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$

Answer

$\operatorname{cosec}\theta = \frac{13}{12} \quad \Rightarrow \sin\theta = \frac{12}{13} \ \ and \ \cos\theta = \frac{5}{13}$

$$now, \frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \frac{2\times\frac{12}{13} - 3\times\frac{5}{13}}{4\times\frac{12}{13} - 9\times\frac{5}{13}}$$

$$= \frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} - \frac{45}{13}}$$

$$= \frac{\frac{24-15}{13}}{\frac{48-45}{13}} = \frac{\frac{9}{13}}{\frac{3}{13}} = \frac{9}{13}\times\frac{13}{3} = 3$$

$\operatorname{cosec}\theta = \frac{13}{12}$

12. Question

If $\sin\theta + \cos\theta = \sqrt{2}\cos(90^0 - \theta)$, find $\cot\theta$.

Answer

$$\sin\theta + \cos\theta = \sqrt{2}\cos(90^0 - \theta)$$
$$\cos\theta = \sqrt{2}\sin\theta - \sin\theta$$
$$\cos\theta = \left(\sqrt{2} - 1\right)\sin\theta$$
$$\frac{\cos\theta}{\sin\theta} = \sqrt{2} - 1$$

CCE - Formative Assessment

1. Question

Define an identity.

Answer

An equation that is true for all values of the variables involved is said to be an identity. For example:

$a^2 - b^2 = (a - b)(a + b)$

$\sin^2\theta + \cos^2\theta = 1$

2. Question

What is the value of $(1 - \cos^2\theta)\operatorname{cosec}^2\theta$?

Answer

To find: $(1 - \cos^2\theta)\operatorname{cosec}^2\theta$

$\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$\therefore \operatorname{cosec}^2\theta = \frac{1}{\sin^2\theta}$

$\Rightarrow (1 - \cos^2\theta)\operatorname{cosec}^2\theta = (1 - \cos^2\theta)\frac{1}{\sin^2\theta}$ ....(i)

$\because \sin^2\theta + \cos^2\theta = 1$

$\therefore \sin^2\theta = 1 - \cos^2\theta$

$\Rightarrow$ from (i), we have

$$(1 - \cos^2\theta)\operatorname{cosec}^2\theta = \sin^2\theta\frac{1}{\sin^2\theta} = 1$$

3. Question

What is the value of $(1 + \cot^2\theta)\sin^2\theta$?

Answer

To find: $(1 + \cot^2\theta) \sin^2\theta$

$\because 1 + \cot^2 \theta = \mathrm{cosec}^2 \theta$

$\therefore (1 + \cot^2\theta) \sin^2\theta = \mathrm{cosec}^2 \theta \sin^2 \theta$

Also, $\mathrm{cosec}\,\theta = \frac{1}{\sin\theta}$

$\Rightarrow \mathrm{cosec}^2\theta = \frac{1}{\sin^2\theta}$

$\Rightarrow (1 + \cot^2\theta)\sin^2\theta = \mathrm{cosec}^2\theta \sin^2\theta = \frac{1}{\sin^2\theta}\sin^2\theta = 1$

4. Question

What is the value of □$\sin^2\theta + \frac{1}{1+\tan^2\theta}$?

Answer

To find: $\sin^2\theta + \frac{1}{1+\tan^2\theta}$

$\because 1 + \tan^2 \theta = \sec^2 \theta$

$\therefore \sin^2\theta + \frac{1}{1+\tan^2\theta} = \sin^2\theta + \frac{1}{\sec^2\theta}$

Also, we know that $\cos\theta = \frac{1}{\sec\theta}$

$\Rightarrow \cos^2\theta = \frac{1}{\sec^2\theta}$

$\Rightarrow \sin^2\theta + \frac{1}{1+\tan^2\theta} = \sin^2\theta + \frac{1}{\sec^2\theta} = \sin^2\theta + \cos^2\theta$

Also,

$\because \sin^2 \theta + \cos^2 \theta = 1$

$\therefore \sin^2\theta + \frac{1}{1+\tan^2\theta} = \sin^2\theta + \frac{1}{\sec^2\theta} = \sin^2\theta + \cos^2\theta = 1$

5. Question

If $\sec \theta + \tan\theta = x$, write the value of $\sec \theta - \tan \theta$ in terms of x.

Answer

Given: $\sec \theta + \tan \theta = x$ ....(i)

To find: $\sec \theta - \tan \theta$

We know that $1 + \tan^2 \theta = \sec^2 \theta$

$\Rightarrow 1 = \sec^2 \theta - \tan^2 \theta$

Now, $\because a^2 - b^2 = (a - b)(a + b)$

$\Rightarrow 1 = \sec^2 \theta - \tan^2 \theta = (\sec \theta - \tan \theta)(\sec \theta + \tan \theta)$

$\Rightarrow$ From (i), we have

$1 = (\sec \theta - \tan \theta)\, x$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{x}$

6. Question

If $\mathrm{cosec}\,\theta - \cot \theta = \alpha$, write the value of $\mathrm{cosec}\,\theta + \cot \alpha$.

Answer

Given: cosec θ – cot θ = α ....(i)

To find: cosec θ + cot θ

We know that $1 + \cot^2\theta = \text{cosec}^2\theta$

$\Rightarrow 1 = \text{cosec}^2\theta - \cot^2\theta$

Now, $\because a^2 - b^2 = (a - b)(a + b)$

$\Rightarrow 1 = \text{cosec}^2\theta - \cot^2\theta = (\text{cosec}\,\theta - \cot\theta)(\text{cosec}\,\theta + \cot\theta)$

⇒ From (i), we have

1 = α (cosec θ + cot θ)

$\Rightarrow \text{cosec}\,\theta + \cot\theta = \frac{1}{\alpha}$

7. Question

Write the value of $\text{cosec}^2(90° - \theta) - \tan^2\theta$.

Answer

To find: $\text{cosec}^2(90° - \theta) - \tan^2\theta$

∵ cosec (90° – θ) = sec θ

$\therefore \text{cosec}^2(90° - \theta) = \sec^2\theta$

$\Rightarrow \text{cosec}^2(90°\ \theta) - \tan^2\theta = \sec^2\theta - \tan^2\theta$

Now, $\because 1 + \tan^2\theta = \sec^2\theta$

$\therefore \text{cosec}^2(90° - \theta) - \tan^2\theta = \sec^2\theta - \tan^2\theta$

$= 1 + \tan^2\theta - \tan^2\theta = 1$

8. Question

Write the value of sin A cos (90° – A) + cos A sin (90° – A).

Answer

To find: sin A cos (90° – A) + cos A sin (90° – A)

∵ cos (90° – A) = sin A and sin (90° – A) = cos A ....(i)

∴ sin A cos (90° – A) + cos A sin (90° – A)

= sin A sin A + cos A cos A [Using (i)]

$= \sin^2 A + \cos^2 A$

Now, $\because \sin^2\theta + \cos^2\theta = 1$

∴ sin A cos (90° – A) + cos A sin (90° – A)

$= \sin^2 A + \cos^2 A = 1$

9. Question

Write the value of $\cot^2\theta - \frac{1}{\sin^2\theta}$.

Answer

To find: $\cot^2\theta - \frac{1}{\sin^2\theta}$

$\because \text{cosec}\,\theta = \frac{1}{\sin\theta}$

$\Rightarrow\ \text{cosec}^2\theta = \dfrac{1}{\sin^2\theta}$

$\Rightarrow\ \cot^2\theta - \dfrac{1}{\sin^2\theta} = \cot^2\theta - \text{cosec}^2\theta$

Also, we know that $1 + \cot^2\theta = \text{cosec}^2\theta$

$\Rightarrow \cot^2\theta - \text{cosec}^2\theta = -1$

$\Rightarrow\ \cot^2\theta - \dfrac{1}{\sin^2\theta} = \cot^2\theta - \text{cosec}^2\theta = -1$

10. Question

If x = a sin θ and y = b cos θ, what is the value of $b^2x^2 + a^2y^2$?

Answer

Given: x = a sin θ and y = b cos θ

$\Rightarrow x^2 = a^2 \sin^2\theta$ and $y^2 = b^2 \cos^2\theta$ ....(i)

To find: $b^2x^2 + a^2y^2$

Consider $b^2x^2 + a^2y^2 = b^2 a^2 \sin^2\theta + a^2 b^2 \cos^2\theta$

$= a^2 b^2 (\sin^2\theta + \cos^2\theta)$

$= a^2 b^2 (1)$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$= a^2 b^2$

11. Question

If $\sin\theta = \dfrac{4}{5}$, what is the value of cot θ + cosec θ?

Answer

Given: $\sin\theta = \frac{4}{5}$

To find: cot θ + cosec θ

$\because \sin^2\theta + \cos^2\theta = 1$

$\therefore \cos^2\theta = 1 - \sin^2\theta$

$\Rightarrow \cos\theta = \sqrt{1 - \sin^2\theta}$

$\Rightarrow \cos\theta = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{25 - 16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

Now, as $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{3/5}{4/5} = \frac{3}{4}$

Also, $\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{1}{4/5} = \frac{5}{4}$

$\Rightarrow \cot\theta\ +\ \text{cosec}\,\theta = \dfrac{3}{4} + \dfrac{5}{4} = \dfrac{3\ +\ 5}{4} = \dfrac{8}{4} = 2$

12. Question

What is the value of $9\cot^2\theta - 9\,\text{cosec}^2\theta$?

Answer

To find: $9\cot^2\theta - 9\,\text{cosec}^2\theta$

Consider $9 \cot^2 \theta - 9 \operatorname{cosec}^2 \theta = 9 (\cot^2 \theta - \operatorname{cosec}^2 \theta)$

Now $\because 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

$\Rightarrow \cot^2 \theta - \operatorname{cosec}^2 \theta = -1$

$\Rightarrow 9 \cot^2 \theta - 9 \operatorname{cosec}^2 \theta = 9 (\cot^2 \theta - \operatorname{cosec}^2 \theta) = 9(-1) = -9$

13. Question

What is the value of $6\tan^2\theta - \frac{6}{\cos^2\theta}$?

Answer

To find: $6\tan^2\theta - \frac{6}{\cos^2\theta}$

$\because \sec\theta = \frac{1}{\cos\theta}$

$\Rightarrow \sec^2\theta = \frac{1}{\cos^2\theta}$

$\Rightarrow 6\tan^2\theta - \frac{6}{\cos^2\theta} = 6\tan^2\theta - 6\sec^2\theta = 6(\tan^2\theta - \sec^2\theta)$

Now, as $1 + \tan^2 \theta = \sec^2 \theta$

$\Rightarrow \tan^2 \theta - \sec^2 \theta = -1$

$\Rightarrow 6\tan^2\theta - \frac{6}{\cos^2\theta} = 6(\tan^2\theta - \sec^2\theta) = 6(-1) = -6$

14. Question

What is the value of $\frac{\tan^2\theta - \sec^2\theta}{\cot^2\theta - \operatorname{cosec}^2\theta}$?

Answer

To find: $\frac{\tan^2\theta - \sec^2\theta}{\cot^2\theta - \operatorname{cosec}^2\theta}$

We know that $1 + \tan^2 \theta = \sec^2 \theta$

And $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

$\Rightarrow \tan^2 \theta - \sec^2 \theta = -1$

And $\cot^2 \theta - \operatorname{cosec}^2 \theta = -1$

$\Rightarrow \frac{\tan^2\theta - \sec^2\theta}{\cot^2\theta - \operatorname{cosec}^2\theta} = \frac{-1}{-1} = 1$

15. Question

What is the value of $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$?

Answer

To find: $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$

$\because (a - b)(a + b) = a^2 - b^2$

$\therefore (1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$

$= (1 + \tan^2\theta)(1 - \sin^2\theta)$

Now, as $\sin^2 \theta + \cos^2 \theta = 1$

$\Rightarrow 1 - \sin^2 \theta = \cos^2 \theta$ ....(i)

Also, we know that $1 + \tan^2 \theta = \sec^2 \theta$ ....(ii)

Using (i) and (ii), we have

$(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$

$= (1 + \tan^2\theta)(1 - \sin^2\theta)$

$= \sec^2\theta \cos^2\theta$

$\because \sec\theta = \frac{1}{\cos\theta}$

$\Rightarrow \sec^2\theta = \frac{1}{\cos^2\theta}$

$\Rightarrow (1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$

$= \sec^2\theta \cos^2\theta$

$= \frac{1}{\cos^2\theta}\cos^2\theta = 1$

16. Question

If $\cos A = \frac{7}{25}$, find the value of tan A + cot A.

Answer

Given: $\cos A = \frac{7}{25}$

To find: tan A + cot A

$\because \sin^2 A + \cos^2 A = 1$

$\Rightarrow \sin^2 A = 1 - \cos^2 A$

$\Rightarrow \sin A = \sqrt{1 - \cos^2 A}$

$$= \sqrt{1 - \left(\frac{7}{25}\right)^2} = \sqrt{1 - \frac{49}{625}} = \sqrt{\frac{625 - 49}{625}} = \sqrt{\frac{576}{625}} = \frac{24}{25}$$

Now, as $\tan A = \frac{\sin A}{\cos A} = \frac{24/25}{7/25} = \frac{24}{7}$

And $\cot A = \frac{1}{\tan A} = \frac{7}{24}$

$$\Rightarrow \tan A + \cot A = \frac{24}{7} + \frac{7}{24} = \frac{576 + 49}{168} = \frac{625}{168}$$

17. Question

If $\sin\theta = \frac{1}{3}$, then find the value of $2\cot^2\theta + 2$.

Answer

Given: $\sin\theta = \frac{1}{3}$

To find: The value of $2\cot^2\theta + 2$.

Solution: $\sin\theta = \frac{1}{3}$

$\because \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{1/3} = 3$

$\Rightarrow \operatorname{cosec}^2\theta = 3^2 = 9$

Also, $1 + \cot^2 \theta = \text{cosec}^2 \theta$

$\Rightarrow \cot^2 \theta = \text{cosec}^2 \theta - 1 = 9 - 1 = 8$

$\Rightarrow 2 \cot^2 \theta + 2 = 2 (8) + 2 = 16 + 2 = 18$Hence, the value of $2 \cot^2 \theta + 2$ is 18.

18. Question

If $\cos\theta = \frac{3}{4}$, then find the value of $9 \tan^2\theta + 9$.

Answer

Given: $\cos\theta = \frac{3}{4}$

To find: $9 \tan^2 \theta + 9$

$\because \sec\theta = \frac{1}{\cos\theta} = \frac{4}{3}$

$\therefore \sec^2\theta = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$

Also, we know that $1 + \tan^2 \theta = \sec^2 \theta$

$\Rightarrow \tan^2\theta = \sec^2\theta - 1 = \frac{16}{9} - 1 = \frac{16-9}{9} = \frac{7}{9}$

$\Rightarrow 9\tan^2\theta + 9 = 9\left(\frac{7}{9}\right) + 9 = 7 + 9 = 16$

19. Question

If $\sec^2\theta (1 + \sin \theta) (1 - \sin \theta) = k$, then find the value of k.

Answer

Given: $\sec^2\theta (1 + \sin \theta) (1 - \sin \theta) = k$

To find: k

Consider $\sec^2\theta (1 + \sin \theta) (1 - \sin \theta)$

$\because (a - b) (a + b) = a^2 - b^2$

$\therefore \sec^2\theta (1 + \sin \theta) (1 - \sin \theta) = \sec^2 \theta (1 - \sin^2 \theta)$

Now, as $\sin^2 \theta + \cos^2 \theta = 1$

$\Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$

$\Rightarrow \sec^2 \theta (1 + \sin \theta) (1 - \sin \theta) = \sec^2 \theta (1 - \sin^2 \theta)$

$= \sec^2 \theta \cos^2 \theta$

Now, $\because \sec\theta = \frac{1}{\cos\theta}$

$\Rightarrow \sec^2\theta = \frac{1}{\cos^2\theta}$

$\Rightarrow \sec^2 \theta (1 + \sin \theta) (1 - \sin \theta) = \sec^2 \theta (1 - \sin^2 \theta)$

$= \sec^2 \theta \cos^2 \theta$

$= \frac{1}{\cos^2\theta}\cos^2\theta = 1$

$\Rightarrow k = 1$

20. Question

If $\text{cosec}^2\theta (1 + \cos \theta) (1 - \cos \theta) = \lambda$, then find the value of λ.

Answer

Given: $cosec^2\theta$ (1 + cos θ) (1 − cos θ) = λ

To find: λ

Consider $cosec^2\theta$ (1 + cos θ) (1 − cos θ)

∵ (a – b) (a + b) = $a^2 - b^2$

∴ $cosec^2\theta$ (1 + cos θ) (1 − cos θ) = $cosec^2\theta$ (1 – $cos^2\theta$)

Now, as $sin^2\theta + cos^2\theta = 1$

⇒ $sin^2\theta = 1 - cos^2\theta$

⇒ $cosec^2\theta$ (1 + cos θ) (1 − cos θ) = $cosec^2\theta$ (1 – $cos^2\theta$)

= $cosec^2\theta\ sin^2\theta$

Now, ∵ $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

⇒ $\operatorname{cosec}^2\theta = \frac{1}{\sin^2\theta}$

⇒ $cosec^2\theta$ (1 + cos θ) (1 − cos θ) = $cosec^2\theta$ (1 – $cos^2\theta$)

= $cosec^2\theta\ sin^2\theta$

$= \frac{1}{\sin^2\theta}\sin^2\theta = 1$

21. Question

If $sin^2\theta\ cos^2\theta$ (1 + $tan^2\theta$) (1 + $cot^2\theta$) = λ, then find the value of λ.

Answer

Given: $sin^2\theta\ cos^2\theta$ (1 + $tan^2\theta$) (1 + $cot^2\theta$) = λ

To find: λ

We know that $1 + tan^2\theta = sec^2\theta$

And $1 + cot^2\theta = cosec^2\theta$

⇒ $sin^2\theta\ cos^2\theta$ (1 + $tan^2\theta$) (1 + $cot^2\theta$)

= $sin^2\theta\ cos^2\theta\ sec^2\theta\ cosec^2\theta$

Now, ∵ $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

⇒ $\operatorname{cosec}^2\theta = \frac{1}{\sin^2\theta}$

And ∵ $\sec\theta = \frac{1}{\cos\theta}$

⇒ $\sec^2\theta = \frac{1}{\cos^2\theta}$

⇒ $sin^2\theta\ cos^2\theta$ (1 + $tan^2\theta$) (1 + $cot^2\theta$)

= $sin^2\theta\ cos^2\theta\ sec^2\theta\ cosec^2\theta$

$= \sin^2\theta\cos^2\theta\frac{1}{\cos^2\theta}\frac{1}{\sin^2\theta} = 1$

⇒ λ = 1

22. Question

If 5x = sec θ and $\frac{5}{x} = \tan\theta$, find the value of $5\left(x^2 - \frac{1}{x^2}\right)$.

Answer

Given: 5x = sec θ

$\Rightarrow x = \frac{\sec\theta}{5}$

$\Rightarrow x^2 = \frac{\sec^2\theta}{25}$ ....(i)

And $\frac{5}{x} = \tan\theta$

$\Rightarrow x = \frac{5}{\tan\theta}$

$\Rightarrow x^2 = \frac{25}{\tan^2\theta}$

$\Rightarrow \frac{1}{x^2} = \frac{\tan^2\theta}{25}$ ....(ii)

To find: $5\left(x^2 - \frac{1}{x^2}\right)$

Consider $5\left(x^2 - \frac{1}{x^2}\right) = 5\left(\frac{\sec^2\theta}{25} - \frac{1}{x^2}\right)$ [Using (i)]

$= 5\left(\frac{\sec^2\theta}{25} - \frac{\tan^2\theta}{25}\right)$ [Using (ii)]

$= 5\left(\frac{\sec^2\theta - \tan^2\theta}{25}\right) = \frac{1}{5}(\sec^2\theta - \tan^2\theta)$

Now, as 1 + $\tan^2$ θ = $\sec^2$ θ

⇒ 1 = $\sec^2$ θ - $\tan^2$ θ

$\Rightarrow 5\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{5}(\sec^2\theta - \tan^2\theta) = \frac{1}{5}$

23. Question

If cosec θ = 2x and $\cot\theta = \frac{2}{x}$, find the value of $2\left(x^2 - \frac{1}{x^2}\right)$

Answer

Given: cosec θ = 2x

$\Rightarrow x = \frac{\operatorname{cosec}\theta}{2}$

$\Rightarrow x^2 = \frac{\operatorname{cosec}^2\theta}{4}$ ....(i)

And $\cot\theta = \frac{2}{x}$

$\Rightarrow x = \frac{2}{\cot\theta}$

$\Rightarrow x^2 = \frac{4}{\cot^2\theta}$

$\Rightarrow \frac{1}{x^2} = \frac{\cot^2\theta}{4}$ ....(ii)

To find: $2\left(x^2 - \frac{1}{x^2}\right)$

Consider $2\left(x^2 - \frac{1}{x^2}\right) = 2\left(\frac{\text{cosec}^2\theta}{4} - \frac{1}{x^2}\right)$ [Using (i)]

$= 2\left(\frac{\text{cosec}^2\theta}{4} - \frac{\cot^2\theta}{4}\right)$ [Using (ii)]

$$= 2\left(\frac{\text{cosec}^2\theta - \cot^2\theta}{4}\right) = \frac{1}{2}\left(\text{cosec}^2\theta - \cot^2\theta\right)$$

Now, as $1 + \cot^2\theta = \text{cosec}^2\theta$

$\Rightarrow 1 = \text{cosec}^2\theta - \cot^2\theta$

$$\Rightarrow 2\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{2}\left(\text{cosec}^2\theta - \cot^2\theta\right) = \frac{1}{2}$$

1. Question

If sec θ + tan θ = x, then sec θ =

A. $\frac{x^2+1}{x}$

B. $\frac{x^2+1}{2x}$

C. $\frac{x^2-1}{2x}$

D. $\frac{x^2-1}{x}$

Answer

Given: sec θ + tan θ = x ....(i)

To find: sec θ

We know that $1 + \tan^2\theta = \sec^2\theta$

$\Rightarrow \sec^2\theta - \tan^2\theta = 1$

$\because a^2 - b^2 = (a - b)(a + b)$

$\therefore \sec^2\theta - \tan^2\theta = (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

⇒ From (i), we have

$\Rightarrow (\sec\theta - \tan\theta)\, x = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{x}$ ....(ii)

Adding (i) and (ii), we get

$$\sec\theta + \sec\theta = x + \frac{1}{x}$$

$$\Rightarrow \sec\theta = \frac{x^2 + 1}{2x}$$

$$\Rightarrow \sec\theta = \frac{x^2 + 1}{2x}$$

2. Question

If sec θ + tan θ = x, then tan θ =

A. $\frac{x^2+1}{x}$

B. $\frac{x^2-1}{x}$

C. $\frac{x^2+1}{2x}$

D. $\frac{x^2-1}{2x}$

Answer

Given: sec θ + tan θ = x ....(i)

To find: tan θ

We know that $1 + \tan^2\theta = \sec^2\theta$

$\Rightarrow \sec^2\theta - \tan^2\theta = 1$

$\because a^2 - b^2 = (a - b)(a + b)$

$\therefore \sec^2\theta - \tan^2\theta = (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

⇒ From (i), we have

$\Rightarrow (\sec\theta - \tan\theta)\, x = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{x}$ ....(ii)

Subtracting (ii) from (i), we get

$$\tan\theta + \tan\theta = x - \frac{1}{x}$$

$$\Rightarrow 2\tan\theta = \frac{x^2-1}{x}$$

$$\Rightarrow \tan\theta = \frac{x^2-1}{2x}$$

3. Question

$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ is equal to

A. sec θ + tan θ

B. sec θ − tan θ

C. $\sec^2\theta + \tan^2\theta$

D. $\sec^2\theta - \tan^2\theta$

Answer

Note: Since all the options involve the trigonometric ratios sec θ and tan θ, so we divide the whole term (numerator as well as denominator) by cos θ.

To find: $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$

Consider $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$

Dividing numerator and denominator by cos θ, we get

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\frac{\frac{1+\sin\theta}{\cos\theta}}{\frac{1-\sin\theta}{\cos\theta}}} = \sqrt{\frac{\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}}$$

$$= \sqrt{\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}} \left[\because \sec\theta = \frac{1}{\cos\theta} \text{ and } \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

Rationalizing the term by multiplying it by $\sqrt{\sec\theta+\tan\theta}$,

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\frac{\frac{1+\sin\theta}{\cos\theta}}{\frac{1-\sin\theta}{\cos\theta}}} = \sqrt{\frac{\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}} = \sqrt{\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}}$$

$$= \sqrt{\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}} \times \sqrt{\frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}}$$

$$= \sqrt{\frac{(\sec\theta+\tan\theta)^2}{\sec^2\theta-\tan^2\theta}}$$

Now, as $1 + \tan^2\theta = \sec^2\theta$

$\Rightarrow \sec^2\theta - \tan^2\theta = 1$

$$\Rightarrow \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\frac{(\sec\theta+\tan\theta)^2}{\sec^2\theta-\tan^2\theta}} = \sqrt{(\sec\theta+\tan\theta)^2} = \sec\theta+\tan\theta$$

4. Question

The value of $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$ is

A. cot θ − cosec θ

B. cosec θ + cot θ

C. $\text{cosec}^2\theta + \cot^2\theta$

D. $(\cot\theta + \text{cosec}\,\theta)^2$

Answer

Note: Since all the options involve the trigonometric ratios cosec θ and cot θ, so we divide the whole term (numerator as well as denominator) by sin θ.

To find: $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$

Consider $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$

Dividing numerator and denominator by sin θ, we get

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}}} = \sqrt{\frac{\text{cosec}\,\theta+\cot\theta}{\text{cosec}\,\theta-\cot\theta}} \left[\because \text{cosec}\,\theta = \frac{1}{\sin\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

Rationalizing the term by multiplying it by $\sqrt{\text{cosec}\,\theta+\cot\theta}$,

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}}} = \sqrt{\frac{\operatorname{cosec}\theta+\cot\theta}{\operatorname{cosec}\theta-\cot\theta}}$$

$$= \sqrt{\frac{\operatorname{cosec}\theta+\cot\theta}{\operatorname{cosec}\theta-\cot\theta}} \times \sqrt{\frac{\operatorname{cosec}\theta+\cot\theta}{\operatorname{cosec}\theta+\cot\theta}}$$

$$= \sqrt{\frac{(\operatorname{cosec}\theta+\cot\theta)^2}{\operatorname{cosec}^2\theta-\cot^2\theta}}$$

Now, as $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\Rightarrow \operatorname{cosec}^2\theta - \cot^2\theta = 1$

$$\Rightarrow \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{(\operatorname{cosec}\theta+\cot\theta)^2}{\operatorname{cosec}^2\theta-\cot^2\theta}} = \sqrt{(\operatorname{cosec}\theta+\cot\theta)^2}$$

$$= \operatorname{cosec}\theta + \cot\theta$$

5. Question

$\sec^4 A - \sec^2 A$ is equal to

A. $\tan^2 A - \tan^4 A$

B. $\tan^4 A - \tan^2 A$

C. $\tan^4 A + \tan^2 A$

D. $\tan^2 A + \tan^4 A$

Answer

Note: Since all the options involve the trigonometric ratio tan θ, so we use the identity $1 + \tan^2\theta = \sec^2\theta$.

To find: $\sec^4 A - \sec^2 A$

Consider $\sec^4 A - \sec^2 A = (\sec^2 A)^2 - \sec^2 A$

Now, as $\sec^2 A = 1 + \tan^2 A$

$\Rightarrow \sec^4 A - \sec^2 A = (\sec^2 A)^2 - \sec^2 A$

$= (1 + \tan^2 A)^2 - (1 + \tan^2 A)$

$= 1 + \tan^4 A + 2\tan^2 A - 1 - \tan^2 A$

$= \tan^4 A + \tan^2 A$

6. Question

$\cos^4 A - \sin^4 A$ is equal to

A. $2\cos^2 A + 1$

B. $2\cos^2 A - 1$

C. $2\sin^2 A - 1$

D. $2\sin^2 A + 1$

Answer

To find: $\cos^4 A - \sin^4 A$

Consider $\cos^4 A - \sin^4 A = (\cos^2 A)^2 - (\sin^2 A)^2$

$\because a^2 - b^2 = (a - b)(a + b)$

$\therefore \cos^4 A - \sin^4 A = (\cos^2 A)^2 - (\sin^2 A)^2$

$= (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)$

$= (\cos^2 A - \sin^2 A)$ [$\because \cos^2 A + \sin^2 A = 1$]

$= \cos^2 A - (1 - \cos^2 A)$ [$\because \sin^2 A = 1 - \cos^2 A$]

$= \cos^2 A - 1 + \cos^2 A = 2\cos^2 A - 1$

7. Question

$\frac{\sin\theta}{1+\cos\theta}$ is equal to

A. $\frac{1+\cos\theta}{\sin\theta}$

B. $\frac{1-\cos\theta}{\cos\theta}$

C. $\frac{1-\cos\theta}{\sin\theta}$

D. $\frac{1-\sin\theta}{\cos\theta}$

Answer

To find: $\frac{\sin\theta}{1+\cos\theta}$

Consider $\frac{\sin\theta}{1+\cos\theta}$

Rationalizing the above fraction by (1 − cos θ),

$$\frac{\sin\theta}{1+\cos\theta} = \frac{\sin\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta} = \frac{\sin\theta\,(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$= \frac{\sin\theta\,(1-\cos\theta)}{(1-\cos^2\theta)}$ [$\because (a - b)(a + b) = a^2 - b^2$]

$\because \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$

$$\Rightarrow \frac{\sin\theta}{1+\cos\theta} = \frac{\sin\theta\,(1-\cos\theta)}{(1-\cos^2\theta)} = \frac{\sin\theta\,(1-\cos\theta)}{\sin^2\theta} = \frac{1-\cos\theta}{\sin\theta}$$

8. Question

$\frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta}$ is equal to

A. 0

B. 1

C. sin θ + cos θ

D. sin θ − cos θ

Answer

Given: $\frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta}$

To find: The value of $\frac{\sin\theta}{1-\cot\theta}+\frac{\cos\theta}{1-\tan\theta}$

Solution:

Use:

$$\tan\theta=\frac{sin\theta}{\cos\theta}, \cot\theta=\frac{cos\theta}{sin\theta}$$

So,

$$\frac{\sin\theta}{1-\cot\theta}+\frac{\cos\theta}{1-\tan\theta}=\frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}=\frac{\sin\theta}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\cos\theta}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$=\frac{\sin^2\theta}{\sin\theta-\cos\theta}+\frac{\cos^2\theta}{\cos\theta-\sin\theta}$$

$$=\frac{\sin^2\theta}{\sin\theta-\cos\theta}-\frac{\cos^2\theta}{\sin\theta-\cos\theta}$$

$$=\frac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}$$

Using the identity,

$a^2 - b^2 = (a - b)(a + b)$

$$\frac{\sin\theta}{1-\cot\theta}+\frac{\cos\theta}{1-\tan\theta}=\frac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}=\frac{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}{\sin\theta-\cos\theta}$$
$$=\sin\theta+\cos\theta$$

9. Question

The value of $(1 + \cot\theta - \text{cosec}\,\theta)(1 + \tan\theta + \sec\theta)$ is

A. 1

B. 2

C. 4

D. 0

Answer

To find: $(1 + \cot\theta - \text{cosec}\,\theta)(1 + \tan\theta + \sec\theta)$

Consider $(1 + \cot\theta - \text{cosec}\,\theta)(1 + \tan\theta + \sec\theta)$

$$=\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)$$

$$\left[\because \cot\theta=\frac{\cos\theta}{\sin\theta}, \text{cosec}\,\theta=\frac{1}{\sin\theta}, \tan\theta=\frac{\sin\theta}{\cos\theta}, \sec\theta=\frac{1}{\cos\theta}\right]$$

$$=\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)$$
$$=\left(\frac{(\sin\theta+\cos\theta)-1}{\sin\theta}\right)\left(\frac{(\sin\theta+\cos\theta)+1}{\cos\theta}\right)$$

$=\frac{(\sin\theta+\cos\theta)^2-1}{\sin\theta\cos\theta}$ [$\because (a - b)(a + b) = a^2 - b^2$]

$$=\frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$

$=\frac{1+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$$=\frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}=2$$

10. Question

$\frac{\tan\theta}{\sec\theta-1}+\frac{\tan\theta}{\sec\theta+1}$ is equal to

A. 2 tan θ

B. 2 sec θ

C. 2 cosec θ

D. 2 tan θ sec θ

Answer

To find: $\frac{\tan\theta}{\sec\theta-1}+\frac{\tan\theta}{\sec\theta+1}$

Consider $\frac{\tan\theta}{\sec\theta-1}+\frac{\tan\theta}{\sec\theta+1}$

$\because \tan\theta=\frac{\sin\theta}{\cos\theta}, \sec\theta=\frac{1}{\cos\theta}$

$\therefore \frac{\tan\theta}{\sec\theta-1}+\frac{\tan\theta}{\sec\theta+1}=\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}-1}+\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}+1}=\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1-\cos\theta}{\cos\theta}}+\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cos\theta}}$

$=\frac{\sin\theta}{1-\cos\theta}+\frac{\sin\theta}{1+\cos\theta}$

$=\frac{\sin\theta(1+\cos\theta)+\sin\theta(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$=\frac{\sin\theta+\sin\theta\cos\theta+\sin\theta-\sin\theta\cos\theta}{(1-\cos^2\theta)}$

$=\frac{2\sin\theta}{\sin^2\theta}$ [$\because \sin^2\theta = 1 - \cos^2\theta$]

$=\frac{2}{\sin\theta}$

$=2\operatorname{cosec}\theta \left[\because \operatorname{cosec}\theta=\frac{1}{\sin\theta}\right]$

11. Question

(cosec θ − sin θ) (sec θ − cos θ) (tan θ + cot θ) is equal

A. 0

B. 1

C. −1

D. None of these

Answer

To find: (cosec θ – sin θ) (sec θ – cos θ) (tan θ + cot θ)

$\because \operatorname{cosec}\theta=\frac{1}{\sin\theta}, \sec\theta=\frac{1}{\cos\theta}, \tan\theta=\frac{\sin\theta}{\cos\theta}, \cot\theta=\frac{\cos\theta}{\sin\theta}$

∴ (cosec θ – sin θ) (sec θ – cos θ) (tan θ + cot θ)

$=\left(\frac{1}{\sin\theta}-\sin\theta\right)\left(\frac{1}{\cos\theta}-\cos\theta\right)\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)$

$=\left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}\right)$

Now, as $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$

And $\cos^2\theta = 1 - \sin^2\theta$

$$\Rightarrow \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right) = \left(\frac{\cos^2\theta}{\sin\theta}\right)\left(\frac{\sin^2\theta}{\cos\theta}\right)\left(\frac{1}{\sin\theta\cos\theta}\right) = 1$$

Hence the answer is 'B'

12. Question

If $x = a\cos\theta$ and $y = b\sin\theta$, then $b^2x^2 + a^2y^2 =$

A. a^2b^2

B. ab

C. a^4b^4

D. $a^2 + b^2$

Answer

Given: $x = a\sin\theta$ and $y = b\cos\theta$

$\Rightarrow x^2 = a^2\sin^2\theta$ and $y^2 = b^2\cos^2\theta$ ....(i)

To find: $b^2x^2 + a^2y^2$

Consider $b^2x^2 + a^2y^2 = b^2a^2\sin^2\theta + a^2b^2\cos^2\theta$

$= a^2b^2(\sin^2\theta + \cos^2\theta)$

$= a^2b^2(1)$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= a^2b^2$

13. Question

If $x = a\sec\theta$ and $y = b\tan\theta$, then $b^2x^2 - a^2y^2 =$

A. ab

B. $a^2 - b^2$

C. $a^2 + b^2$

D. a^2b^2

Answer

Given: $x = a\sec\theta$ and $y = b\tan\theta$

$\Rightarrow x^2 = a^2\sec^2\theta$ and $y^2 = b^2\tan^2\theta$ ....(i)

To find: $b^2x^2 - a^2y^2$

Consider $b^2x^2 - a^2y^2 = b^2a^2\sec^2\theta - a^2b^2\tan^2\theta$

$= a^2b^2(\sec^2\theta - \tan^2\theta)$

$= a^2b^2(1)$ $[\because \sec^2\theta - \tan^2\theta = 1]$

$= a^2b^2$

14. Question

$\frac{\cot\theta}{\cot\theta - \cot 3\theta} + \frac{\tan\theta}{\tan\theta - \tan 3\theta}$ is equal to

A. 0

B. 1

C. −1

D. 2

Answer

To find: $\frac{\cot\theta}{\cot\theta-\cot3\theta}+\frac{\tan\theta}{\tan\theta-\tan3\theta}$

Consider $\frac{\cot\theta}{\cot\theta-\cot3\theta}+\frac{\tan\theta}{\tan\theta-\tan3\theta}$

$$=\frac{\cot\theta(\tan\theta-\tan3\theta)+\tan\theta(\cot\theta-\cot3\theta)}{(\cot\theta-\cot3\theta)(\tan\theta-\tan3\theta)}$$

$$=\frac{\cot\theta\tan\theta-\cot\theta\tan3\theta+\tan\theta\cot\theta-\tan\theta\cot3\theta}{\cot\theta\tan\theta-\cot\theta\tan3\theta-\cot3\theta\tan\theta+\cot3\theta\tan3\theta}$$

$$=\frac{1-\cot\theta\tan3\theta+1-\tan\theta\cot3\theta}{1-\cot\theta\tan3\theta-\cot3\theta\tan\theta+1}\left[\because\cot\theta=\frac{1}{\tan\theta}\Rightarrow\cot\theta\tan\theta=1\right]$$

$$=\frac{2-\cot\theta\tan3\theta-\tan\theta\cot3\theta}{2-\cot\theta\tan3\theta-\tan\theta\cot3\theta}=1$$

15. Question

$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta)$ is equal to

A. 0

B. 1

C. −1

D. None of these

Answer

To find: $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta)$

First, we consider

$\sin^6\theta + \cos^6\theta = (\sin^2\theta)^3 + (\cos^2\theta)^3$

Now, as $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$\Rightarrow a^3 + b^3 = (a + b)^3 - 3a^2b - 3ab^2$

$\Rightarrow \sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)^3 - 3(\sin^2\theta)^2\cos^2\theta - 3\sin^2\theta(\cos^2\theta)^2$

$= 1 - 3\sin^4\theta\cos^2\theta - 3\sin^2\theta\cos^4\theta$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$= 1 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$

$= 1 - 3\sin^2\theta\cos^2\theta$ [$\because \sin^2\theta + \cos^2\theta = 1$] ....(i)

Next, we consider

$\sin^4\theta + \cos^4\theta = (\sin^2\theta)^2 + (\cos^2\theta)^2$

Now, as $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + b^2 = (a + b)^2 - 2ab$

$\Rightarrow \sin^4\theta + \cos^4\theta$

$= (\sin^2\theta)^2 + (\cos^2\theta)^2$

$= (\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta$

$= 1 - 2 \sin^2 \theta \cos^2 \theta$ [$\because \sin^2 \theta + \cos^2 \theta = 1$] ....(ii)

Now, using (i) and (ii), we have

$2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta)$

$= 2(1 - 3 \sin^2 \theta \cos^2 \theta) - 3(1 - 2 \sin^2 \theta \cos^2 \theta)$

$= 2 - 6 \sin^2 \theta \cos^2 \theta - 3 + 6 \sin^2 \theta \cos^2 \theta$

$= 2 - 3 = - 1$

16. Question

If a cos θ + b sin θ and a sin θ − b cos θ = 3, then $a^2 + b^2$ =

A. 7

B. 12

C. 25

D. None of these

Answer

Given: $a \cos \theta + b \sin \theta = 4$

Squaring both sides, we get

$(a \cos \theta + b \sin \theta)^2 = 4^2$

$\Rightarrow a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \sin \theta \cos \theta = 16$ ....(i)

and $a \sin \theta - b \cos \theta = 3$

Squaring both sides, we get

$(a \sin \theta - b \cos \theta)^2 = 3^2$

$\Rightarrow a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta = 9$ ....(ii)

To find: $a^2 + b^2$

Adding (i) and (ii), we get

$a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \sin \theta \cos \theta$

$+ a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta = 16 + 9$

$\Rightarrow a^2 (\sin^2 \theta + \cos^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta) = 25$

$\Rightarrow a^2 + b^2 = 25$ [$\because \sin^2 \theta + \cos^2 \theta = 1$]

17. Question

If a cot θ + b cosec θ = p and b cot θ + a cosec θ = q, then $p^2 - q^2$ =

A. $a^2 - b^2$

B. $b^2 - a^2$

C. $a^2 + b^2$

D. b − a

Answer

Given: $a \cot \theta + b \operatorname{cosec} \theta = p$

Squaring both sides, we get

$(a \cot \theta + b \operatorname{cosec} \theta)^2 = p^2$

$\Rightarrow a^2 \cot^2 \theta + b^2 \text{cosec}^2 \theta + 2ab \cot \theta \text{ cosec } \theta = p^2$ ......(i)

and $b \cot \theta + a \text{ cosec } \theta = q$

Squaring both sides, we get

$(b \cot \theta + a \text{ cosec } \theta)^2 = q^2$

$\Rightarrow b^2 \cot^2 \theta + a^2 \text{cosec}^2 \theta + 2ab \cot \theta \text{ cosec } \theta = q^2$ ......(ii)

To find: $p^2 - q^2$

Subtracting (ii) from (i), we get

$a^2 \cot^2 \theta + b^2 \text{cosec}^2 \theta + 2ab \cot \theta \text{ cosec } \theta - b^2 \cot^2 \theta - a^2 \text{cosec}^2 \theta - 2ab \cot \theta \text{ cosec } \theta = p^2 - q^2$

$\Rightarrow P^2 - q^2 = a^2 (\cot^2 \theta - \text{cosec}^2 \theta) + b^2 (\text{cosec}^2 \theta - \cot^2 \theta)$

$= a^2 (- 1) + b^2 (1)$ [$\because 1 = \text{cosec}^2 \theta - \cot^2 \theta$]

$= b^2 - a^2$

18. Question

The value of $\sin^2 29° + \sin^2 61°$ is

A. 1

B. 0

C. $2 \sin^2 29°$

D. $2 \cos^2 61$

Answer

To find: $\sin^2 29° + \sin^2 61°$

Consider $\sin^2 29° + \sin^2 61°$

$\because 29 = 90 - 61$

$\therefore \sin^2 29° + \sin^2 61° = \sin^2 (90° - 61°) + \sin^2 61°$

Now, as $\sin (90° - \theta) = \cos \theta$

$\Rightarrow \sin^2 29° + \sin^2 61° = \sin^2 (90° - 61°) + \sin^2 61°$

$= \cos^2 61° + \sin^2 61°$

$= 1$ [$\sin^2 \theta + \cos^2 \theta = 1$]

19. Question

If $x = r \sin \theta \cos \varphi$, $y = r \sin \theta \sin \varphi$ and $z = r \cos \theta$, then

A. $x^2 + y^2 + z^2 = r^2$

B. $x^2 + y^2 - z^2 = r^2$

C. $x^2 - y^2 + z^2 = r^2$

D. $z^2 + y^2 - x_2 = r^2$

Answer

Given: $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \varphi$ and $z = r \cos \theta$,

Solution:$x = r \sin \theta \cos \phi$

Squaring both sides, we get

$x^2 = r^2 \sin^2 \theta \cos^2\phi$ ....(i)

and $y = r \sin \theta \sin \phi$

Squaring both sides, we get

⇒ $y^2 = r^2 \sin^2 \theta \sin^2\phi$ ....(ii)

$z = r \cos \theta$Squaring both sides, we get

⇒ $z^2 = r^2 \cos^2 \theta$ ....(iii)

Adding (i), (ii) and (iii), we get

$x^2 + y^2 + z^2 = r^2 \sin^2 \theta \cos^2\phi + r^2 \sin^2 \theta \sin^2\phi + r^2 \cos^2 \theta$

$= r^2 (\sin^2 \theta \cos^2\phi + \sin^2 \theta \sin^2\phi + \cos^2 \theta)$

$= r^2 [\sin^2 \theta (\cos^2\phi + \sin^2\phi) + \cos^2 \theta]$

$\because \sin^2 \theta + \cos^2 \theta = 1$

$= r^2 [\sin^2 \theta + \cos^2 \theta]$

Again apply the identity $\sin^2 \theta + \cos^2 \theta = 1$

$= r^2$ Hence $x^2 + y^2 + z^2 = r^2$

20. Question

If $\sin \theta + \sin^2 = 1$, then $\cos^2\theta + \cos^4\theta =$

A. −1

B. 1

C. 0

D. None of these

Answer

Given: $\sin \theta + \sin^2 \theta = 1$

⇒ $\sin \theta = 1 - \sin^2 \theta = \cos^2 \theta$ [$\because \sin^2 \theta + \cos^2 \theta = 1$]......(i)

⇒ $\text{Sin}^2 \theta = (\cos^2 \theta)^2 = \cos^4 \theta$ ......(ii)

To find: $\cos^2 \theta + \cos^4 \theta$

Consider $\cos^2 \theta + \cos^4 \theta = \sin \theta + \sin^2 \theta$ [Using (i) and (ii)]

= 1 [Given]

21. Question

If $a \cos \theta + b \sin \theta = m$ and $a \sin \theta - b \cos \theta = n$, then $a^2 + b^2 =$

A. $m^2 - n^2$

B. m^2n^2

C. $n^2 - m^2$

D. $m^2 + n^2$

Answer

Given: $a \cos \theta + b \sin \theta = m$

Squaring both sides, we get

$(a \cos \theta + b \sin \theta)^2 = m^2$

⇒ $a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \cos \theta \sin \theta = m^2$ ......(i)

And $a \sin \theta - b \cos \theta = n$

Squaring both sides, we get

$(a \sin \theta - b \cos \theta)^2 = n^2$

$\Rightarrow a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta = n^2$ ......(ii)

To find: $a^2 + b^2$

Adding (i) and (ii), we get

$a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \cos \theta \sin \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta = m^2 + n^2$

$\Rightarrow a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta) = m^2 + n^2$

$\Rightarrow a^2 + b^2 = m^2 + n^2$ [$\because \sin^2 \theta + \cos^2 \theta = 1$]

22. Question

If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A$

A. −1

B. 0

C. 1

D. None of these

Answer

Given: $\cos A + \cos^2 A = 1$

$\Rightarrow \cos A = 1 - \cos^2 A = \sin^2 A$ [$\because \sin^2 A + \cos^2 A = 1$]......(i)

Squaring both sides, we get

$\Rightarrow \cos^2 A = (\sin^2 A)^2 = \sin^4 A$ ......(ii)

To find: $\sin^2 A + \sin^4 A$

Consider $\sin^2 A + \sin^4 A = \cos A + \cos^2 A$ [From (i) and (ii)]

= 1

23. Question

If $x = a \sec \theta \cos \varphi$, $y = b \sec \theta \sin \varphi$ and $z = c \tan \theta$, then $\frac{x^2}{a^2} + \frac{y^2}{b^2} =$

A. $\frac{z^2}{c^2}$

B. $1 - \frac{z^2}{c^2}$

C. $\frac{z^2}{c^2} - 1$

D. $1 + \frac{z^2}{c^2}$

Answer

Given: x a $\sec \theta \cos \phi$

Squaring both sides, we get

$x^2 = a^2 \sec^2 \theta \cos^2\phi$

and $y = b \sec \theta \sin \phi$

Squaring both sides, we get

$y^2 = b^2 \sec^2 \theta \sin^2\phi$

And z = c tan θ

$\Rightarrow z^2 = c^2 \tan^2 \theta$

$\Rightarrow \tan^2\theta = \frac{z^2}{c^2}$ ....(i)

To find: $\frac{x^2}{a^2} + \frac{y^2}{b^2}$

Consider $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{a^2 \sec^2\theta \cos^2\phi}{a^2} + \frac{b^2 \sec^2\theta \sin^2\phi}{b^2}$

$= \sec^2 \theta \cos^2\phi + \sec^2 \theta \sin^2\phi$

$= \sec^2 \theta (\cos^2\phi + \sin^2\phi)$

$= \sec^2 \theta$ [$\because \sin^2\phi + \cos^2\phi = 1$]

$= 1 + \tan^2 \theta$ [$\because 1 + \tan^2 \theta = \sec^2 \theta$]

$= 1 + \frac{z^2}{c^2}$

24. Question

If a cos θ − b sin θ = c, then a sin θ + b cos θ =

A. $\pm\sqrt{a^2 + b^2 + c^2}$

B. $\pm\sqrt{a^2 + b^2 - c^2}$

C. $\pm\sqrt{c^2 - a^2 + b^2}$

D. None of these

Answer

Given: a cos θ – b sin θ = c

To find: a sin θ + b cos θ

Consider a cos θ – b sin θ = c

Squaring both sides, we get

$(a \cos \theta - b \sin \theta)^2 = c^2$

$\because (a - b)^2 = a^2 + b^2 - 2ab$

∴ a cos θ – b sin θ = c

$\Rightarrow a^2 \cos^2 \theta + b^2 \sin^2 \theta - 2ab \sin\theta \cos \theta = c^2$ ......(i)

Now, $\because \sin^2 \theta + \cos^2 \theta = 1$

$\therefore \sin^2 \theta = 1 - \cos^2 \theta$ and $\cos^2 \theta = 1 - \sin^2 \theta$

⇒ From (i), we have

$\Rightarrow a^2 (1 - \sin^2 \theta) + b^2 (1 - \cos^2 \theta) - 2ab \sin \theta \cos \theta = c^2$

$\Rightarrow a^2 - a^2 \sin^2 \theta + b^2 - b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta = c^2$

$\Rightarrow a^2 + b^2 - (a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta) = c^2$

$\Rightarrow - (a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta) = c^2 - a^2 - b^2$

$\Rightarrow a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta = a^2 + b^2 - c^2$

$\Rightarrow (a \sin \theta)^2 + (b \cos \theta)^2 + 2 (a \sin \theta) (b \cos \theta) = a^2 + b^2 - c^2$

$\Rightarrow (a \sin \theta + b \cos \theta)^2 = a^2 + b^2 - c^2$

$\Rightarrow a \sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$

25. Question

$9 \sec^2 A - 9 \tan^2 A$ is equal to

A. 1

B. 9

C. 8

D. 0

Answer

To find: $9 \sec^2 A - 9 \tan^2 A$

Consider $9 \sec^2 A - 9 \tan^2 A = 9 (\sec^2 A - \tan^2 A)$

$\because 1 + \tan^2 A = \sec^2 A$

$\therefore 9 \sec^2 A - 9 \tan^2 A = 9 (\sec^2 A - \tan^2 A)$

$= 9 (1 + \tan^2 A - \tan^2 A) = 9$

26. Question

$(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta) =$

A. 0

B. 1

C. 1

D. −1

Answer

To find: $(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta)$

Consider $(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta)$

$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$

$\left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}, \operatorname{cosec}\theta = \frac{1}{\sin\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta}, \sec\theta = \frac{1}{\cos\theta}\right]$

$= \left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)$

$= \left(\frac{(\sin\theta + \cos\theta) + 1}{\cos\theta}\right)\left(\frac{(\sin\theta + \cos\theta) - 1}{\sin\theta}\right)$

$= \frac{(\sin\theta + \cos\theta)^2 - 1}{\sin\theta\cos\theta}$ $[\because (a - b)(a + b) = a^2 - b^2]$

$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$

$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$

27. Question

$(\sec A + \tan A) (1 - \sin A) =$

A. sec A

B. sin A

C. cosec A

D. cos A

Answer

To find: (sec A + tan A) (1 − sin A)

Consider (sec A + tan A) (1 − sin A)

We know that $\sec A = \frac{1}{\cos A}$ and $\tan A = \frac{\sin A}{\cos A}$

$$\Rightarrow (\sec A + \tan A)(1 - \sin A) = \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A)$$

$\because$ (a + b) (a – b) = $a^2 - b^2$

$$\therefore (\sec A + \tan A)(1 - \sin A) = \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A) = \frac{1 - \sin^2 A}{\cos A}$$

Also, $\sin^2 A + \cos^2 A = 1 \Rightarrow 1 - \sin^2 A = \cos^2 A$

$$\Rightarrow (\sec A + \tan A)(1 - \sin A) = \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A$$

28. Question

$\frac{1 + \tan^2 A}{1 + \cot^2 A}$ is equal to

A. $\sec^2 A$

B. −1

C. $\cot^2 A$

D. $\tan^2 A$

Answer

To find: $\frac{1 + \tan^2 A}{1 + \cot^2 A}$

Consider $\frac{1 + \tan^2 A}{1 + \cot^2 A}$

$\because 1 + \tan^2 A = \sec^2 A$ and $1 + \cot^2 A = \text{cosec}^2 A$

$$\therefore \frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A}{\text{cosec}^2 A} = \frac{1/\cos^2 A}{1/\sin^2 A} \left[\because \sec A = \frac{1}{\cos A} \text{ and } \text{cosec } A = \frac{1}{\sin A}\right]$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A \left[\because \tan A = \frac{\sin A}{\cos A}\right]$$